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Time Scale Evaluation of Economic Forecasts

Antonis Michis[1]

Abstract

A maximal overlap discrete wavelet transform is used to obtain time scale decompositions of economic forecasts and their errors. The generated time scale components can be used in loss measures and tests for comparing forecast accuracy to evaluate whether the forecasts accurately capture the cyclical features of the data.

Keywords: forecast accuracy; loss measures; time scales; wavelets

JEL classification: C53; E37

[1] The opinions expressed in this paper are those of the author and do not necessarily reflect the views of the Central Bank of Cyprus or the Eurosystem.

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1. Introduction

Wavelets constitute a relatively new but powerful tool for the analysis of time series that can be used in many economic applications. Yogo (2008) used wavelets to decompose the US GDP into different time scales (or cycles), which permits the identification of the business cycle component in the data. The time scale components generated by wavelets can also be used to examine relationships between economic variables across frequencies. For example, Gallegati et al. (2011) investigated the relationship between wage inflation and unemployment in the US across frequencies and over time.

Wavelets have also been used in econometric estimation and testing. Jensen (2000) and Gilles et al. (2009) examined the use of wavelets in the estimation of models for long memory processes, and Michis and Sapatinas (2007) constructed wavelet instruments to improve the efficiency of GMM estimators. Additional applications of wavelets include testing for serial correlation of unknown form in panel data regression models as suggested by Hong and Kao (2004) and the unit root tests proposed by Fan and Gencay (2010).

In this study, the maximal overlap discrete wavelet transform (MODWT) is used to decompose economic forecasts and their associated forecast errors into different time scales. The generated time scale components capture different cyclical features of the data, which permits an evaluation of forecast accuracy across the cycle. Because the time scale components have equal length with the actual time series, they can be incorporated in standard loss measures (e.g., the mean squared error) and tests for comparing forecast accuracy (e.g., the test proposed by Diebold and Marianno, 1995).

The wavelet time scale decompositions are briefly explained in Section 2. Section 3 demonstrates how the generated time scale components can be used to evaluate forecasts over different cycles, and Section 4 provides an empirical study using four forecasts for shipping volume data.

2. Time scale decompositions of economic time series

Wavelets constitute families of basis functions defined within the set of square integrable functions $L^2(R)$. Time series or functions can be represented by a sequence of projections onto a basis of father (ϕ) and mother (ψ) wavelets that are defined as follows (see Gallegati et. al., 2011):

$$\phi_{j,k} = 2^{-j/2}\phi\left(\frac{t-2^j k}{2^j}\right) \quad \text{and} \quad \psi_{j,k} = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right).$$

A unit decrease in the value of j $\{j = 1,2,3,\ldots,J\}$ expands the range of the mother wavelet proportional to 2^j, which reduces its width and doubles its frequency. In contrast, the father wavelet is not affected by changes in j, but a unit increase in k shifts the location of both the father and mother wavelets.

For a time series, χ_t, with length $N = 2^J$, the wavelet multi-resolution approximation can be expressed as

$$\chi_t = \sum_k s_{J,k}\,\phi_{J,k}(t) + \sum_k d_{J,k}\,\psi_{J,k}(t) + \sum_k d_{J-1,k}\,\psi_{J-1,k}(t) + \ldots + \sum_k d_{1,k}\,\psi_{1,k}(t).$$

The father and mother wavelet coefficients are, respectively:

$$s_{J,k} = \int \chi_t\,\phi_{J,k}(t)\,dt \quad \text{and} \quad d_{j,k} = \int \chi_t\,\psi_{j,k}(t)\,dt.$$

The father wavelet coefficients capture the smooth, low-frequency trend behaviour in the data, and the mother wavelet coefficients capture all high-frequency, short-term deviations from the trend. The wavelet multi-resolution approximation can also be written as

$$\chi_t = S_J + D_J + D_{J-1} + \ldots + D_1$$

with time scale components $S_J = \sum_k s_{J,k}\,\phi_{J,k}(t)$ and $D_j = \sum_k d_{j,k}\,\psi_{j,k}(t)$.

Therefore, the wavelet multi-resolution analysis decomposes the data into J time scales. Time scale components with higher values of j capture long-term cycles in the data, and the time scale component, S_J, associated with the father wavelets, captures the smooth trend behaviour in the data. In contrast, time scales with small values of j capture the high-frequency, short-term cyclical movements in the data.

For a time series with length $512 = 2^9$ weeks, the wavelet multi-resolution analysis provides a decomposition into nine time scales. The elements of the first time scale component (D_1) capture frequency variation over durations of 2 to 4 weeks. The elements of the second component (D_2) capture frequency variation over durations of 4 to 8 weeks, and accordingly, the third component (D_3) is associated with frequency variation over durations of 8 to 16 weeks. This is the case up to level 9.

In this study, a multi-resolution analysis based on the MODWT is used. The MODWT does not provide an exactly orthogonal decomposition of the time series, but it is more efficient than the basic discrete wavelet transform. It also generates time scale components of equal length with the actual time series that can be used in loss measures and tests of forecast accuracy. In practice, the MODWT is computed with a pyramid algorithm that iteratively filters the time series with a high- and a low-pass filter to produce the vectors of wavelet coefficients (see Percival and Walden, 2000, pp. 174).

3. Time scale evaluation of economic forecasts

Several approaches have been proposed in the literature for evaluating the accuracy of economic forecasts. These range from simple loss measures such as the mean squared error (MSE) and the mean absolute error (MAE) to statistical tests for comparing the accuracy of two (see, e.g., Diebold and Mariano, 1995) or more forecasts (see, e.g., Hansen, 2005).

Since the time scale components generated by the MODWT have equal length with the actual time series data, they can be used in loss measures and tests of forecast accuracy to evaluate whether the forecasts accurately capture the cyclical features of the data. To demonstrate this, let $\varepsilon^i_{j,t+h|t} = \chi_{t+h} - \chi^i_{t+h|t}$ be the h-periods ahead forecast errors associated with method i at time t. Rather than evaluating the forecasts, $\chi^i_{t+h|t}$, at the observed sampling rate of the data, the data are first decomposed into different time scales using the MODWT. In a second step, separate forecast errors are formed for each time scale j as follows:

$$\varepsilon^{i}_{j,t+h|t} = D_{j,t+h} - D^{i}_{j,t+h|t} .$$

The time scale component, $D_{j,t+h}$, refers to the actual data (χ_{t+h}), and the time scale component, $D^{i}_{j,t+h|t}$, refers to the forecasts generated with method i at time t ($\chi^{i}_{t+h|t}$). By deriving the forecasts errors associated with each time scale, it is possible to draw conclusions concerning the forecasting accuracy of the method across the cycle. For example, it is useful to know whether a method is more accurate in forecasting short-term than long-term changes in an economic variable. Using the above time scale definition for the errors, it is possible to calculate the MSE and the MAE for each time scale as follows:

$$MSE_{j} = \frac{1}{T}\sum_{t=1}^{T}(\varepsilon^{i}_{j,t+h|t})^{2} \qquad \text{and} \qquad MAE_{j} = \frac{1}{T}\sum_{t=1}^{T}\left|\varepsilon^{i}_{j,t+h|t}\right| .$$

The time scale errors can also be used in the context of the Diebold-Mariano test to compare the accuracy of two competing methods in forecasting specific cycles in the data. Using a squared loss function $L(\varepsilon^{i}_{j,t+h|t}) = (\varepsilon^{i}_{j,t+h|t})^{2}$, the loss differential between forecasting methods a and b at time scale j is defined as $d_{j,t} = L(\varepsilon^{a}_{j,t+h|t}) - L(\varepsilon^{b}_{j,t+h|t})$. The h -periods ahead forecasts are assumed to be repeatedly computed at time periods $t = t_0,...,T$. For a covariance stationary series, the Diebold-Mariano test statistic (S_j) follows the standard normal distribution under the null hypothesis of zero expected loss differential (equal forecast accuracy)

$$S_{j} = \frac{\bar{d}_{j}}{\sqrt{\hat{V}_{\bar{d}_{j}} / T}} \sim N(0,1) .$$

The sample mean loss differential at time scale j is $\bar{d}_j = \sum_{t=t_0}^{T} d_{j,t} / T$. $\hat{V}_{\bar{d}_j} = \sum_{\tau=-M}^{M} \gamma_{d_j}(\tau)$ is an estimator of the variance of the mean loss differential using the sample autocovariance, $\gamma_{\bar{d}_j}$, at displacements τ and where $M = T^{1/3}$ (see Diebold, 2004, p.300).

4. An empirical study

In this section, the proposed methods are applied to the Atlantic East trade lane (cargo) shipping volume forecasts described in Diebold (2004, p. 305-306). In addition to the actual cargo volume data, this dataset consists of 499 2-week ahead volume forecasts generated with two different methods: (i) based on a quantitative model (Quant.) and (ii) based on a judgmental method (Judgme.). Diebold also suggested the use of a regression combination method (Regress.) using a model with MA(1) errors. For the purposes of this study, a fourth method was also considered that consisted of simple averages (Aver.) of the quantitative and judgmental forecasts. Because the time series include 499 observations, the data were padded with the last value of the series to increase their length to 512 (a power of 2), as suggested by Gencay et al. (2002, p. 144). The MODWT was performed with Wavethresh software using the Daubechies least asymmetric family of wavelets with a filter length of 8, which provided good resolution for the data.

The accuracy of each method at each time scale was evaluated using the MSE and MAE loss measures described in Section 3. The results are included in Table 1. The shaded areas in the table indicate the most accurate methods by time scale. Both loss measures indicate the same methods. For example, the results in time scale TS9 that captures the low-frequency, long-term cyclical movements in the data indicate that the quantitative model provided the most accurate forecasts. For time scales TS1 and TS2, which are associated with high frequency, short-term cyclical movements in the data with lengths of 2 to 4 and 4 to 8 months, respectively, the regression combination method provided the best results. For time scales TS3 (8 to 16 weeks) and TS4 (16 to 32 weeks), the simple averaging method was the most accurate.

Table 1 Loss measures by time scale

Time-scale	Measure	Quant.	Judgme.	Aver.	Regres.
TS9	MSE	**1.959**	256.709	61.594	3.104
TS8	MSE	**0.926**	22.552	5.979	1.166
TS7	MSE	**1.015**	12.947	3.787	3.828
TS6	MSE	**3.839**	12.337	4.723	15.566
TS5	MSE	**4.335**	8.497	4.476	8.394
TS4	MSE	4.898	6.034	**3.878**	6.627
TS3	MSE	6.072	5.179	**4.109**	4.612
TS2	MSE	5.028	3.908	3.348	**2.900**
TS1	MSE	4.287	2.818	2.670	**2.149**
TS9	MAE	**1.239**	15.980	7.793	1.606
TS8	MAE	**0.834**	4.701	2.316	0.931
TS7	MAE	**0.806**	3.319	1.670	1.676
TS6	MAE	**1.483**	2.780	1.664	3.136
TS5	MAE	**1.637**	2.302	1.768	2.341
TS4	MAE	1.730	1.888	**1.553**	1.967
TS3	MAE	1.906	1.781	**1.605**	1.744
TS2	MAE	1.780	1.585	1.490	**1.382**
TS1	MAE	1.673	1.369	1.326	**1.191**

The best methods identified by time scale in Table 1 were also compared against the other methods using the Diebold-Mariano test. The generated p-values are included in Table 2. With the exception of time scale TS5, the null hypothesis of zero expected loss differential was rejected in all time scales, confirming the superiority of the methods included in column 2. In time scale TS5 the quantitative model was found to be equal to the averaging method. Consequently, if the main purpose of the forecasting exercise is to predict short-term movements in cargo shipping volume, then the regression combination method should be used. In a different case, if the goal is to predict the long-term cyclical movements in the series, then the quantitative model should be used.

Table 2 Diebold-Mariano test p-values by time scale

Time scale	Best method	Quant.	Judgme.	Aver.	Regres.
TS9	Quant.	-	0.001	0.001	0.001
TS8	Quant.	-	0.001	0.001	0.003
TS7	Quant.	-	0.001	0.001	0.001
TS6	Quant.	-	0.001	0.096	0.001
TS5	Quant.	-	0.001	0.378*	0.001
TS4	Aver.	0.017	0.001	-	0.001
TS3	Aver.	0.001	0.006	-	0.030
TS2	Regres.	0.001	0.001	0.004	-
TS1	Regres.	0.001	0.001	0.001	-

*Equal predictive accuracy.

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