Fiscal consolidation: Dr Pangloss meets Mr Keynes*

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Abstract

A simple dynamic framework is used to show how consolidation plans that are robust and effective at capacity output can be undermined by demand failure. If the market panics and interest rates rise, the process can indeed become dynamically unstable. Tightening fiscal policy to reassure financial markets can lead to a low level "consolidation trap", however. Better that the Central Bank acts to keep interest rates low; and that fiscal consolidation efforts be state contingent – allowing room for economic stabilisation. The pro-cyclicality of fiscal policy could also be reduced if, as Shiller has argued, debt amortization were state contingent, being indexed to GDP.

Keywords: Debt; Deficits; Fiscal Consolidation; Economic Stabilisation

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"All is for the best in the best of all possible worlds."
Dr Pangloss in *Candide* by Voltaire 1759

Introduction

Public sector debt and deficits in the European community, previously disciplined by the provisions of the Maastricht Treaty, were thrown into disarray by the financial crisis of 2008/9. To restore order to public sector finances, Europe has put in train programmes of Fiscal Consolidation - mandatory for those countries in receipt of emergency Troika support. As the effects of Fiscal Consolidation have proved controversial - both in terms of the stated objectives and in terms of economic growth - we analyse how such programmes are supposed to work; and how they may go awry. For this purpose we use a stripped-down model of two differential equations, describing the dynamics of *debt accumulation* and of *fiscal consolidation,* operating in two regimes – with and without 'multiplier effects' of expenditure reduction.

David Hendry has argued that, in a progressive research paradigm for econometrics, one's results should not just fit the data, but should also account for why previous investigators got things wrong, Davidson et al. (1978). Perhaps the same applies in the policy arena? In this spirit, , the analysis in section 1 begins with an economy where aggregate output matches its potential level at all times and fiscal adjustment along lines envisaged by the Maastricht Treaty has purely *allocative* effects with no impact on the level of activity . In this case we find that a simple error correction policy for expenditure adjustment provides stable approach to equilibrium so long as the speed of adjustment satisfies a minimum condition; and that the process is remarkably robust to variations in the level of debt and the cost of debt service. Attention is drawn, however, to a political constraint - what Ghosh et al. (2011, 2012) refer to as *fiscal fatigue* – which may slowdown the process of adjustment.

As a matter of national income accounting sectoral financial surpluses add up to zero[1]. So, for the government to move in to financial surplus in order to reduce its debt, this requires the private sector to move into financial deficit – leaving aside the adjustment on the external account. If the private sector fails to do this, possibly because it is also trying to deleverage – the result will be a contraction of aggregate demand and a fall of output below capacity, *ceteris paribus*.

[1] $Y = C + I + G \rightarrow Y - C - T - I = S - I = G - T$

Ideally one would study how plans for deleveraging by public and private sectors might interact to determine the level of output. Instead we simply assume that public expenditure cuts have Keynesian multiplier effects: and, in section 2, the process of fiscal consolidation is modified to take into account these *income* effects, which will of course affect tax yields too. To study how fiscal dynamics are affected by the endogeneity of taxes, we work with a 'reduced form' where variables are measured relative to capacity, i.e. they are cyclically adjusted. In simulations, however, the analysis is extended to track the evolution of debt and spending relative to endogenous income, as is customary in macro modelling, see for example Holland and Portes (2012) and Eyraud and Weber (2013), recent papers considered further below.

In section 3 we use the endogenous tax framework to examine cases where fiscal consolidation plans do ultimately succeed in achieving the fiscal targets at full employment; but only after inducing recession. In 3.1 we find that if private sector demand is reasonably resilient - in the sense that there will be no shortfall *even when public expenditure has been cut as needed to achieve the fiscal targets* - then consolidation programmes described earlier will remain reasonably efficient even when adverse multiplier effects are taken in to account.

But things are different if, at the inception of the policy, the economy is at capacity level and demand is being sustained by public spending in excess of what is appropriate for long-run sustainability (because the private sector is also deleveraging, for example). In that case, a policy that promptly removes the prop will face obvious problems, as tax receipts fall, pushing debt levels up rather than down[2]. A period of 'waiting and hoping' for private demand to recover will not only provide disappointing results for debt and deficits, it will involve costly losses of output, as we see in Section 3.2. The effect of 'front loading' consolidation - cutting public expenditure before private sector demand recovers - illustrates why ill-timed fiscal consolidation has been called 'self-defeating austerity', Holland and Portes (2012).

If such income effects throw initial plans for fiscal consolidation off target, the

[2] In practice, as Martin Wolf (2013b) observes, "fiscal tightening…removed a still desperately needed offset to the contractionary forces emanating from crisis-hit private sectors."

authorities may be tempted to step in and correct the policy, typically by a tightening of the fiscal stance. But this may exacerbate the situation, as Eyraud and Weber (2013) point out in an IMF paper on 'the challenge of debt reduction with fiscal consolidation'. They warn of a possible 'negative feedback loop' where the short-run increase in the ratio of debt to income leads the authorities to tighten fiscal policy in order to get the ratio down, "setting off a vicious cycle of slow growth, deflation and further tightening". One solution they suggest is that of cyclically adjusting the debt income ratio; but in section 3.3 we find that, even with cyclically-adjusted debt targets, the risk of getting caught in such a 'consolidation trap' remains, particularly when countries are competing to prove their credit worthiness, as in Europe today. Ironically, it appears that governments which initiate fiscal consolidation to avoid the adverse growth effects of high debt described by Reinhart and Rogoff (2010), may succeed in generating precisely the outcomes they are afraid of!

The fear that higher interest rates could cause the programme of consolidation to unravel is real enough, however. When multiplier effects are at work, interest rates do not have to rise far to cause dynamic instability, as shown in section 4. There is a real possibility of self-fulfilling financial crises as *financial panic* drives up sovereign spreads, along the lines indicated by Calvo (1988). In section 4.1 we discuss how the ECB can head this off, as in the programme of Outright Monetary Transactions (OMT) initiated in 2012.

But what if interest rates do increase, so the system becomes dynamically unstable? Even then, there remains a unique stable path that leads to a sustainable equilibrium. In Section 4.2 we briefly examine steps that might be taken to put the economy on this path, with particular reference to Emerging Market Economies trying to consolidate in the face of exorbitant rates on local currency sovereign debt. These will involve debt write-downs and/or further fiscal tightening. The promise of debt sustainability in these circumstances is essentially a snare and delusion: far better, we argue, that interest rates be brought down. Though sometimes described a policy of "financial repression", cutting excessive interest may lead to economic recovery - with beneficial effects on income distribution if they represent monopoly rents for local banks.

In section 5, two policy alternatives are briefly explored. First, to stabilise the

economy before consolidating debt, i.e. 'back-loading' the consolidation programme and allowing debt to increase as public spending mitigates current recession - with subsequent tax increases to cover the extra debt interest, as in DeLong and Summers (2012). Finally we discuss the notion of swapping public sector debt for equity (state-contingent debt contracts) as proposed by Robert Shiller (1993, 2003) and explored in detail by Barr et al. (2012). To solve the first-mover problems facing financial innovations of this sort, we outline the idea of an official Special Purpose Vehicle (SPV). Section 6 concludes.

1. Fiscal Consolidation as reallocation of capacity output

The principal avowed purpose of fiscal consolidation is to ensure debt sustainability for the public sector. This may be achieved in a variety of ways, such as:

- reducing expenditure and/or raising tax rates;
- debt reduction via inflation or explicit repudiation;
- financial repression, i.e. lowering the rate of interest paid;
- increasing the growth rate
- a debt equity swap

The main focus in this paper is on the reduction of public expenditure to obtain sustainability, as indicated with reference to Figure 1. This shows the level of debt, b, on the vertical axis, and the rates of government expenditure, g, and tax revenue, θ, on the horizontal, *all expressed relative to capacity output*. Assume that that rb is the cost of debt service as a fraction of capacity output, which is growing at the rate γ in real terms and $\gamma + \pi$ in nominal terms, where π is the rate of inflation. Then the dynamics of *bond accumulation* may be written:

$$\text{BA} \qquad \dot{b} = (r - \pi - \gamma)b + g - \theta \tag{1}$$

where the dot notation is used to indicate time derivatives.

The schedule *BB* indicates where the debt/output ratio will remain stationary as the primary surplus, $\theta - g$, is just sufficient to cover the cost of debt service, corrected for growth and inflation, $(r - \pi - \gamma)b$. To the left of *BB,* however, debt will be falling, as indicated by the arrow pointing down from A_0. The converse is true for

expenditure levels labelled A_1 and A_2 to the right of BB, the former with a surplus on the primary budget balance, i.e. $\theta > g$, the latter with a primary deficit. In neither case is the cost of financing the debt covered by a primary surplus, so with the expenditure plus interest charges exceeding the tax base, debt will be growing unsustainably unless action is taken.

It is clear from the Figure that cutting expenditure to point A on the line of stationarity will suffice to stabilise the debt output ratio at b_0. But if b_0 exceeds a desired debt/output target, b^*, then what is needed is a trajectory which takes expenditure to the left of BB for some time until the target is reached at E.

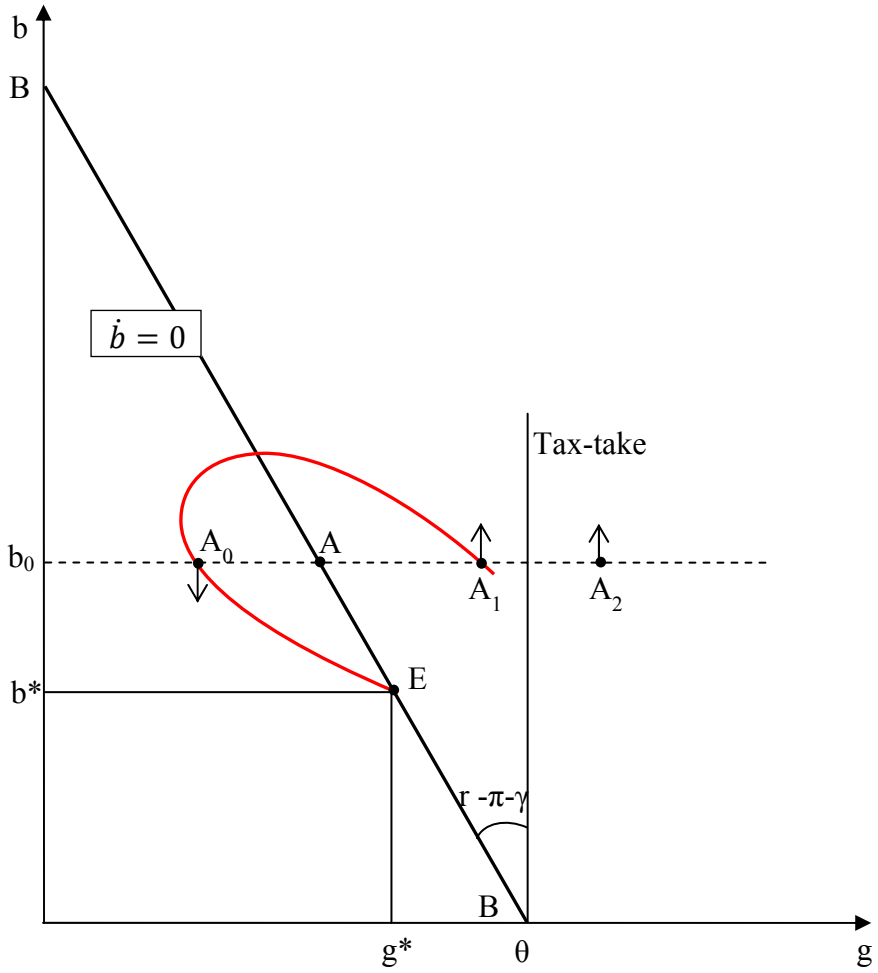

Figure 1. Debt sustainability and government expenditure.

This is the type of consolidation policy we study here on the assumption that it has no effect on the level of capacity or the degree to which it is being used[3]. The broad idea,

[3] Cyclical approach to E is also possible.

as in Barro (1984, chapter 13), is that for forward-looking agents public sector debt is a wash (as it represents future taxes); and shifts in public expenditure which reduce permanent income will be offset by shifts in private consumption.

To avoid an unsustainable build-up of debt, therefore, let spending be cut as long as the structural deficit, $S = rb + g - \theta$, exceeds a target level, δ^*, where S is measured at capacity output and δ^* is chosen[4] to achieve a target steady-state debt/output ratio, b^*, namely $\delta^* = (\gamma+\pi)b^*$. Specifically, let *fiscal consolidation* take the form of error correction, so:

$$\text{FC} \quad \dot{g} = -\alpha(S - \delta^*) = -\alpha(rb + g - \theta - \delta^*) = -\alpha(rb + g - \theta') \tag{2}$$

where $\theta' = \theta + \delta^*$.

The dynamics can be summarised as a two equation linear system, our baseline model of fiscal consolidation:

$$\begin{bmatrix}\dot{g}\\ \dot{b}\end{bmatrix} = \begin{bmatrix}-\alpha & -\alpha r\\ 1 & r - \pi - \gamma\end{bmatrix}\begin{bmatrix}g\\ b\end{bmatrix} + \begin{bmatrix}\alpha\theta'\\ -\theta\end{bmatrix} \tag{3}$$

with adjustment indicated by the arrows in Figure 2, which gives the phase diagram for adjustment under fiscal
consolidation.

[4] So the deficit after adjustment for growth and inflation $S - (\gamma + \pi)b = (r - \pi - \gamma)b + g - \theta$ will be zero when $b = b^*$.

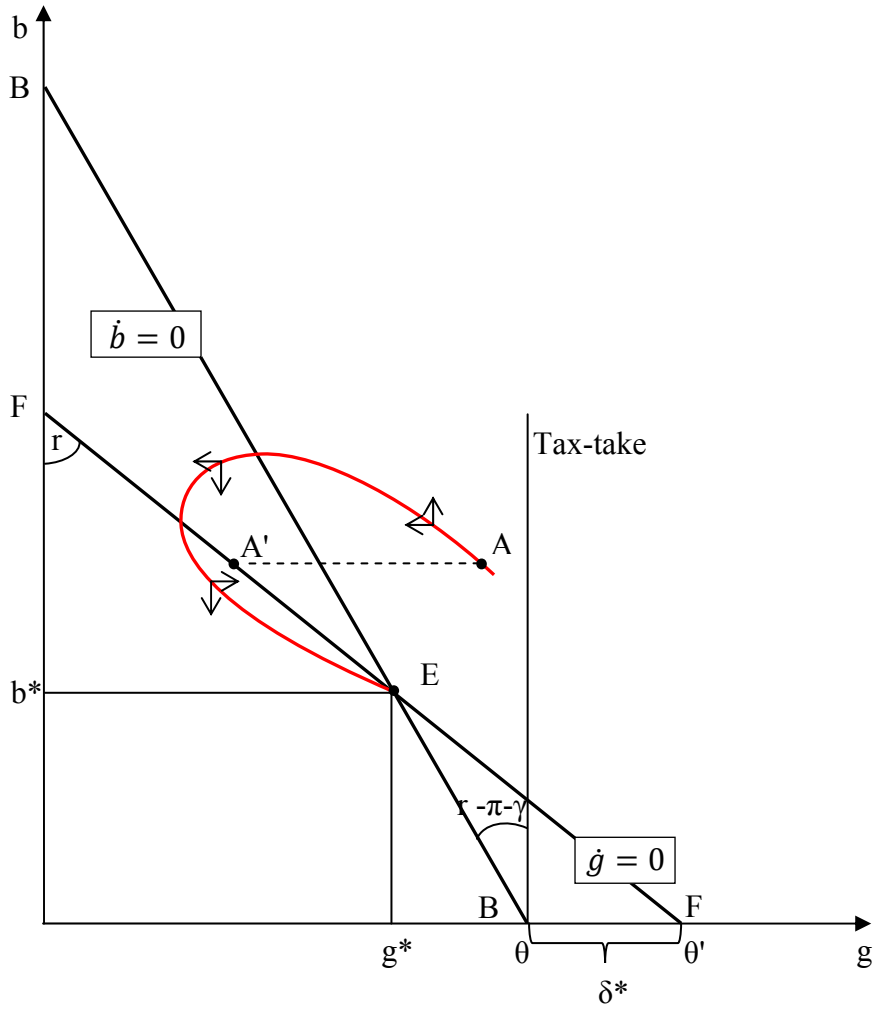

Figure 2. Fiscal consolidation with capacity output: the baseline model.

Central to the dynamics are the lines of stationarity; that for b is, as before, the line BB. The locus of stationarity for g, on the other hand, is indicated by the schedule FF, where the deficit including the real cost of debt service matches the target δ^*. Since, in the steady state at E, $\gamma' b^* = \delta^*$ where $\gamma' = \gamma + \pi$, the target deficit reflects the choice of a desired debt/output ratio - the 60% target of the Maastricht Treaty, for example.

A key parameter here is the speed of consolidation, measured by α. If $\alpha = 0$, the debt/output ratio will increase without limit from a point such as A in Figure 2: with debt unsustainable in the absence of consolidation. With very rapid adjustment, $\alpha \to \infty$, the debt/output ratio will be falling as the system approaches equilibrium E along the schedule FF. For intermediate values of α, there may be cycles: when $\alpha = r - \gamma'$ for example the roots will have no real parts, indicating cycles without convergence.

(Formal details of the dynamics are provided in Appendix A.) On the trajectory shown in the figure starting at point A - drawn on the assumption that the speed of adjustment is sufficient to ensure convergence without cycles - the debt/output ratio will be rising for some time before it reaches a maximum.

The effect of varying α on the pattern of convergence is evident from the simulation results shown in Figure 3 where $r = 0.06,\ \gamma = 0.02,\ \pi = 0.02, \theta = 0.4$ and $\delta^* = 0.024$: as the speed of adjustment slows down, the 'overshooting' of debt increases – and so does the 'undershooting' of government spending. With α of 0.4, for example, debt rises for over 3 years adding about 5% (relative to capacity) before contracting; and government spending falls to a low of 36.5% of capacity before recovery to equilibrium begins 9 years out. But with faster consolidation (α = 0.5), peak debt is reached in 2 years and government spending begins to recover after 7 years.

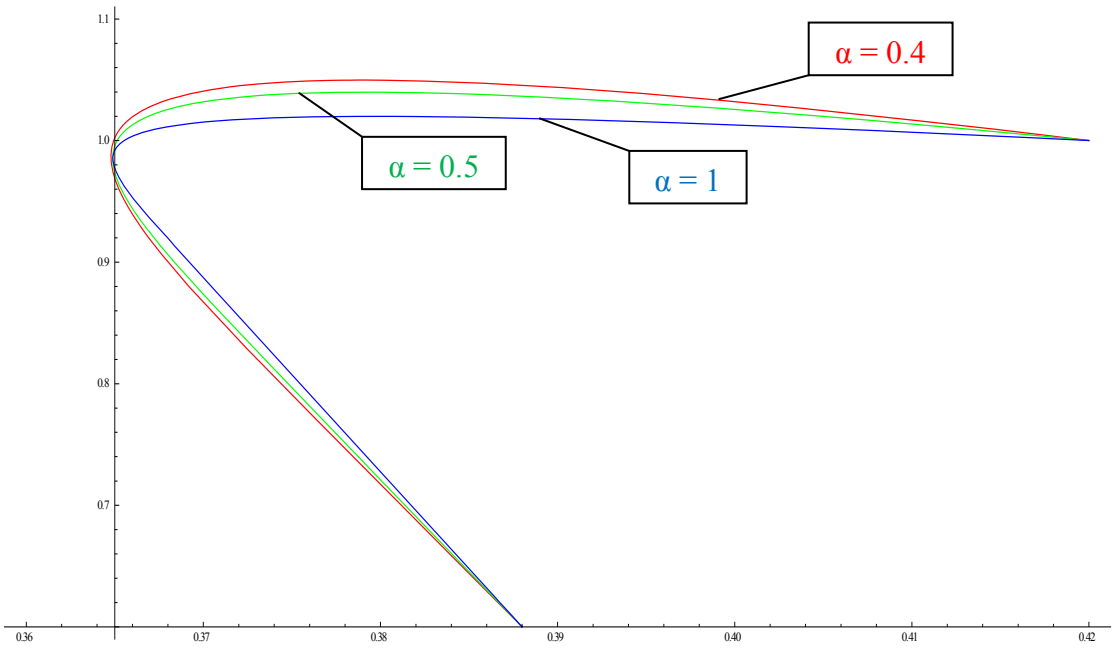

Figure 3. Different speeds of consolidation without cycles.

In the absence of adverse income effects, the policy of fiscal consolidation appears remarkably robust. Convergence to equilibrium is guaranteed no matter what the rate of interest may be, and for a wide range of initial debt levels, so long as the speed of adjustment, α, is greater than the inflation-and-growth-adjusted cost of borrowing, $r - \gamma'$. As far as consolidation goes apparently, "all is for the best for all possible worlds".

This optimistic assessment may be summarised as:

Proposition1: So long as $\alpha > r - \gamma'$, where $\gamma' = \gamma + \pi$, the process of fiscal consolidation will converge to equilibrium; with no cycles if $\alpha > r - \gamma' + 2\sqrt{\alpha\gamma'}$ i.e. $\alpha > r + \gamma' + 2\sqrt{r\gamma'}$.

Proof: see Appendix A.

To avoid any substantial increase in the debt/output ratio and the cost of debt service, the answer seems obvious: to increase the speed of adjustment. What might go wrong?

1.1 Risk of Fiscal Fatigue

Even when there are no income effects from expenditure cuts, the reallocation of resources can prove problematic – involving, as it may, a significant reduction in the supply of public goods. Fiscal adjustment may, in short, be subject to a political 'boundary condition', what Ghosh et al. (2011) call *fiscal fatigue*. Following the formulation proposed by Barr et al. (2012), we assume that this puts *an upper limit on the primary surplus* as a percentage of GDP. So, assuming the tax take is kept at θX, this imposes a lower limit to government expenditure, as shown in Figure 4, where the vertical line at $\underline{g}$ indicates the political limit on expenditure cuts.

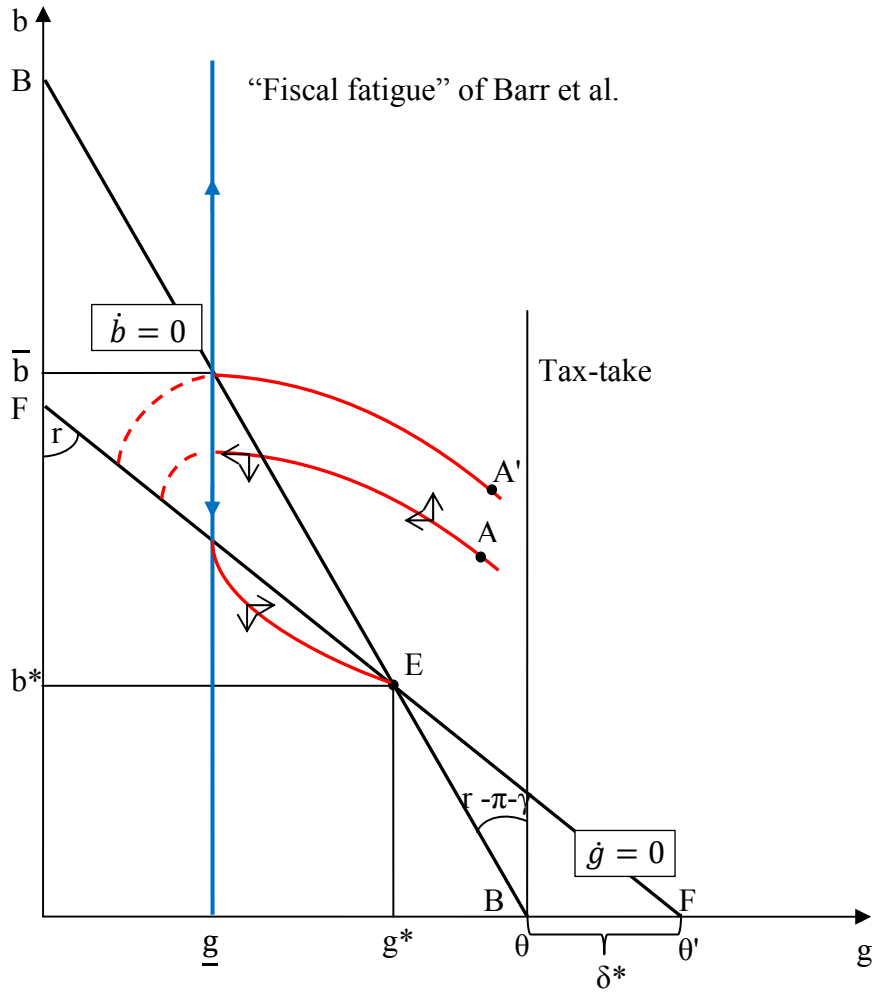

Figure 4. Fiscal fatigue defines an upper debt limit.

Only some trajectories will hit this boundary condition. For those, such as that starting at A, that do and hit at levels below $\bar{b}$, where $g = \underline{g}$ - the debt/output ratio will continue to contract even when government spending stops falling, thanks to the continued expansion of output itself. For those that reach the boundary condition above $\bar{b}$, however, debt would expand without limit - leading ultimately to default of some form[5]. So, as Barr et al. point out, fiscal fatigue sets an upper limit ($\bar{b}$) to sustainable debt.

If Reinhart and Rogoff (2009) are to be believed when they suggest that 90% represents a prudential upper limit for debt (in the sense that beyond that level growth slows to zero), then fiscal fatigue must set in fairly quickly. For if $\bar{b} = 0.9$, and

[5] Barr et al. assume that the sovereign declares default and bond holders suffer a loss given default of 10-30%.

$\theta = 0.4$ and $r - \pi - \gamma = 0.02$, then $\underline{g} = \theta - \overline{b}(r - \pi - \gamma) \approx 0.38$. So, fiscal fatigue would have to set in for spending just two per cent of GDP below this full employment tax-take[6].

Note also that to stabilise the debt/ income ratio, the structural deficit must be positive, $(\gamma+\pi)b^*>0$. Under the Maastricht criteria, for example, where 60% was set as a desirable level for b, then for $\gamma = 0.02$ and $\pi = 0.02$ this would imply a full employment deficit of $60\% \times 0.04 = 2.4\%$ (which lies within the 3% deficit figure permitted by the Treaty).

2. Consolidation with multiplier effects: the endogenous tax model

From the analysis of section 1 it appears that, so long as the starting level of b lies below $\overline{b}$, fiscal consolidation can avoid unsustainable levels of debt simply by choosing an appropriate speed of adjustment (α). This conclusion is, however, based on the optimistic assumption that output is, and remains, at its high employment potential during fiscal consolidation. What if the government is attempting to deleverage at a time when the financial sector is trying to recover from a financial crisis? In these circumstances, the private sector is unlikely to add to demand in the manner suggested by Barro (1984) – quite the contrary, as Richard Koo (2008) points out with respect to the Japanese experience of "balance sheet" recession.

So why is it that governments - in Europe especially - embarked on fiscal consolidation when they did? Martin Wolf (2013b) suggests they took the Greek experience of fiscal profligacy as evidence that high debts elsewhere had the same origins. Thus, he argues:

> the Greek crisis frightened policy makers everywhere. Instead of focusing efforts on remedying the collapse of the financial sector and reducing the overhang of private debt, which were the causes of the crisis, they focused on fiscal deficits. But these were largely a symptom of the crisis, though also, in part, an appropriate policy response to it… in June 2010, shortly after the first Greek programme, leaders of the Group of 20 leading countries, meeting in Toronto, decided to reverse the stimulus, declaring that "advanced economies have committed to fiscal plans that will at least halve deficits by 2013". A sharp tightening followed. Policy makers justified the shift with supportive academic research: the view that fiscal contraction could be expansionary was an

[6] On the 90% debt limit, Martin Wolf (2013a) has remarked that, if true, there would have been no Industrial Revolution in Britain!

encouragement; the view that growth would fall if public debt grew too high was a warning.

This is what we analyse in this section – ill-timed fiscal consolidation where expenditure cuts trigger reductions of income (and taxes) via the multiplier. Recent research in both academia and at the IMF has rehabilitated the multiplier – at least in conditions of recession. Christiano et al. (2011) have gone so far as to suggest that at the zero lower bound for interest rates the multiplier could exceed 3. In their IMF discussion paper, Blanchard and Leigh (2013a, pp.19-20) are much more modest. They note that while the stabilisation efforts implemented in 2009 began with an assumed multiplier of 0.5, the evidence suggest that "actual multipliers it turned out to be substantially above 1 early in the crisis" and from this experience they conclude that "it seems safe for the time being, when thinking about fiscal consolidation, to assume higher multipliers than before the crisis".

An important implication of their findings is that tightening fiscal policy in recession in the name of "fiscal consolidation" will suffer from adverse income effects – so cuts to spending will effectively reduce the tax base in the short-run. It would, of course, be too much to expect IMF officials to challenge the need for fiscal adjustment "in response to elevated debt levels and future pressures on public finances from demographic change", Blanchard and Leigh (2013a, p.20). But their message is clear enough. Efforts for fiscal consolidation should be conditioned on the state of the economy - less now, more later, as they put it in their voxeu piece (Blanchard and Leigh, 2013b).

What if demand matches capacity when the policy begins, but not as cuts proceed - so fiscal consolidation has multiplier effects? One could look explicitly at the evolution of income and of fiscal variables relative to endogenous income, as in Eyraud and Weber (2013), for example. It is, however, analytically more straightforward first to measure things relative to capacity, assumed to be growing exogenously at a constant rate, before measuring things relative to income. The resulting perspective is perhaps that of financial markets, who look at debt and taxes and are less concerned with income levels – unless they threaten long-run growth itself.

Let the plans for fiscal consolidation, with their focus on reducing the structural deficit, continue as before. How robust will they prove when taking account of these

multiplier effects? While the line of stationarity for fiscal spending remains unchanged, the line of stationarity for bonds will now reflect the state of the economy - which will in turn depend on the level of expenditure. How precisely will the schedule *BB* be affected?

Assume that, when the economy is in recession, the spending multiplier is strictly positive, and output can be determined by

$$y = 1 - \mu(\hat{g}_0 - g); \tag{4}$$

where $y = Y/X$ represents aggregate demand relative to capacity, μ is the Keynesian multiplier for public expenditure and $\hat{g}_0$ is the level of government spending that would restore potential output as estimated at the time when fiscal consolidation begins. Allowing for recession, therefore, the bond accumulation equation becomes

$$\dot{b} = (r - \gamma)b + g - \theta y = (r - \pi - \gamma)b + g - \theta[1 - \mu(\hat{g}_0 - g)] \tag{5}$$

which may be rewritten as:

$$\dot{b} = (r - \pi - \gamma)b + g - \theta + \varphi(\hat{g}_0 - g) \tag{6}$$

where $\varphi = \theta\mu$ and $\varphi(\hat{g}_0 - g)$ measures the tax loss.

Thus in graphical terms, the schedule showing stationarity of bonds relative to capacity becomes flatter for lower levels of public spending when $g < \hat{g}_0$: because of the loss of tax base, as long as these recession conditions persist, debt will be less sustainable. The change to the BB schedule is shown by the kink at $g = \hat{g}_0$ in Figures 5 and 7, which also analyse the implications for dynamic adjustment.

Assuming any initial weakness of private sector demand persists, i.e. $\hat{g}_0$ remains unchanged, the addition of multiplier effects from fiscal consolidation therefore implies:

$$\dot{b} = (r - \pi - \gamma)b + (1 - \varphi)g - \theta + \varphi\hat{g}_0 \tag{7}$$

For convenience, we can describe the dynamics of adjustment by adding the rule for fiscal consolidation to obtain a linear system - leaving for later discussion how the dynamics will switch back to those of the baseline model, as and when $g \geq \hat{g}_0$, or when demand spontaneously recovers.

Thus the effect of endogenous taxes is to reduce the slope of the line of stationarity for bonds and to shift the intercept to the left of θ'. Together with the rule for fiscal consolidation discussed earlier, this constitutes the endogenous tax model:

$$\begin{bmatrix}\dot{g}\\ \dot{b}\end{bmatrix} = \begin{bmatrix}-\propto & -\alpha r\\ 1-\phi & r-\pi-\gamma\end{bmatrix}\begin{bmatrix}g\\ b\end{bmatrix} + \begin{bmatrix}\alpha\theta'\\ \phi\hat{g}_0-\theta\end{bmatrix} \quad (8)$$

Subject to switches of regime when $g \geq \hat{g}_0$, this may be solved for long-run steady state values, namely, $b^* = \frac{\delta^*-\phi(\theta'-\hat{g}_0)}{\gamma+\pi-\varphi r}$; and $g^* = \theta' - rb^* = \theta + (r-\gamma-\pi)b^*$.

3. Fiscal consolidation with multiplier effects

How do consolidation plans fare when applied in this context? For ease of comparability we begin with cases where debt and spending reach *the same equilibrium as the full capacity case studied earlier* (see E in Figure 2), so there is no conflict between consolidation and stabilization in the long-run. The first is where the target level of expenditure is itself sufficient to ensure demand meets supply: the second when this is not true, but there is a spontaneous recovery of aggregate demand, as is assumed by DeLong and Summers (2012) in their analysis of fiscal policy in a temporary recession. (An alternative approach, involving a dynamic system of higher order, would be to postulate that the recession conditions die away exponentially, so once again there will be a return to equilibrium at E.)

Under the heading of 'Depression economics: the consolidation trap', we then consider a case of policy failure, where the conflict between consolidation and stabilization is only resolved by adopting tighter targets for the structural deficit (so δ* is reduced).

For simplicity we assume that there are no multiplier effects until the economy moves into recession, when $^{dy}/_{dg} = \mu$. The cases discussed above, where there is effectively no long-run conflict between consolidation and stabilization, can be characterized as follows:

Proposition 2: Assuming that $\alpha - r + \pi + \gamma > 0$ and $\gamma + \pi > \varphi r$, the steady state in the baseline model will be achieved:

1) If $g^*_{Base} = \hat{g}_0$, i.e the level of public expenditure after consolidation will be

sufficient to maintain capacity output; but the trajectory will involve a recession.

2) If $g^*_{Base} < \hat{g}_0$, i.e the level of public expenditure after consolidation will be insufficient to maintain capacity output, and a spontaneous recovery of demand is expected at time T, such that $\hat{g}_T < g_T < g^*_{Base}$; but the trajectory will involve recession until time T.

   Proof: See Appendix B.

3.1 Policy works without a shift in private demand

Given that steady state equilibrium is the same as the baseline, how will the trajectory of debt and spending evolve? We start with the first case where private demand is robust, with recession only when $g < g^*$. This is shown graphically in Figure 5, where recession will only emerge - and the multiplier be relevant - for points to the left of MM. Beginning at a point A, consolidation will shift resources without recession until C is reached and the economy moves into recession. Conceptually, this is the point where Dr Pangloss meets Mr Keynes!

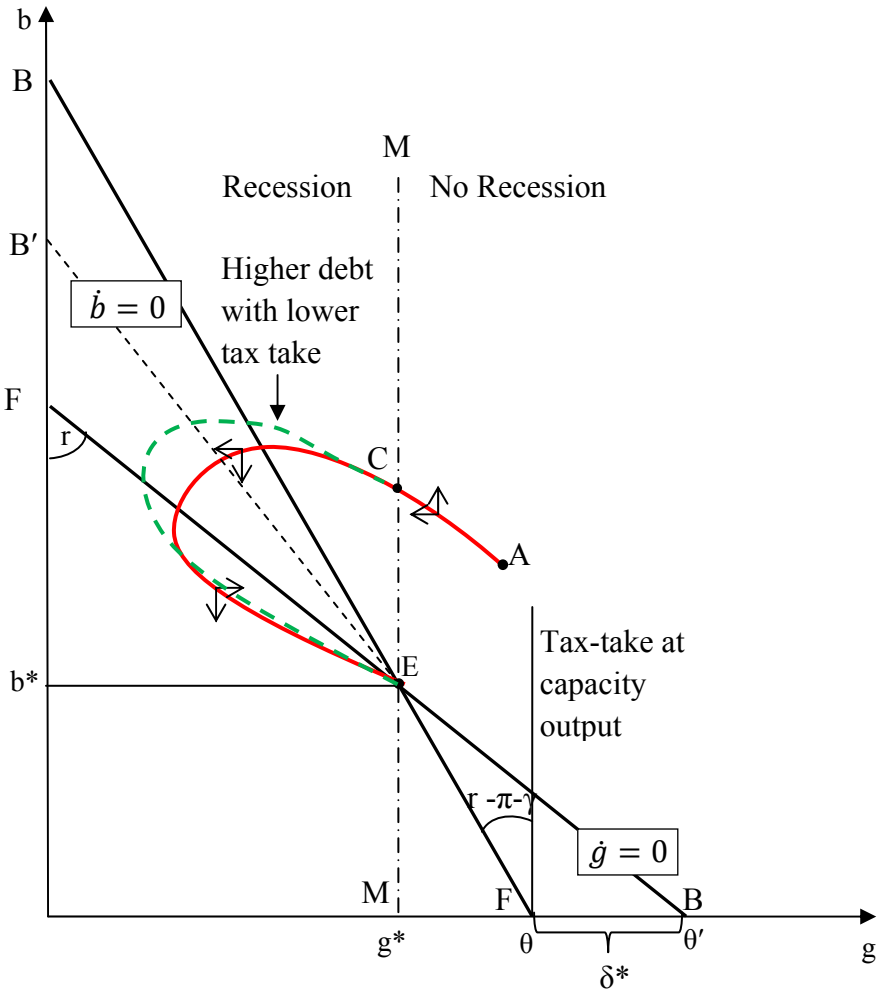

Figure 5. Fiscal stabilisation works, but with temporary recession.

What are the consequences of the demand failure? From C, the relevant trajectory, shown by the dashed line, will depart from that of the baseline model – as the level of debt rises faster as tax receipts fall with income in recession. Thus the results with endogenous income and taxes look rather like the slowing down of fiscal consolidation in the baseline case: debt will rise to a higher peak and spending fall to a lower trough in the consolidation process. But here output will fall below capacity for a while as a result of fiscal consolidation, as the government destabilises the economy in order to stabilise public finances.

Simulations from a point like C where $g^*_{Base} = \hat{g}_0$ are presented in Figure 6. For parameter values where $\varphi = \theta\mu = 0.4 \times 1$, $\delta^* = 0.024$ and $\alpha = 0.6$, the results for debt and spending relative to capacity are quite similar to those in the baseline case. Although income is endogenous, it does not fall greatly in the process of

consolidation; with a multiplier of 1, GDP falls by at most 1.2% of capacity, before returning to “full employment”.

For convenience, the ratios of debt and spending relative to *current GDP* are also shown: this increases the peak debt ratio, and deepens the trough spending/output ratio, which is reached after a period of about 6 years with these parameters.

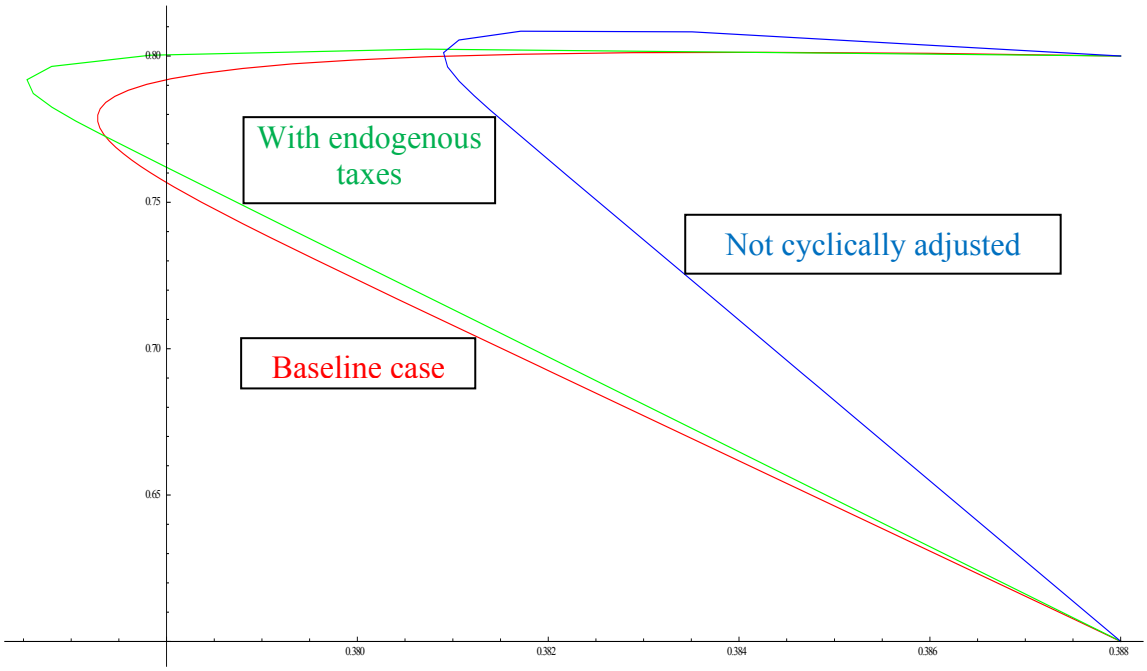

Figure 6. Simulation results which converge to full employment in the long-run.

3.2 Consolidation goes off track, until private demand recovers

The effects of including the multiplier are potentially more dramatic if private demand is - at least initially - insufficient to offset planned consolidation in public expenditure, as in Figure 7, where the line MM, showing the regime switch to recession, lies to the right of E. Starting at point A as before, fiscal consolidation will proceed on track at capacity output until point C is reached. Thereafter, however, it will go off track until there is a recovery of private sector demand.

Assuming the plans for fiscal consolidation remain unchanged, the line of stationarity for bonds will be flatter to the left of MM, which is intersects at point X. Until private sector demand recovers, equilibrium moves from E to E′, as indicated by the dotted line XE′. Along the trajectory leading to this equilibrium, shown by the dashed lines CA′E′, debt is rising and the economy is contracting for some time.

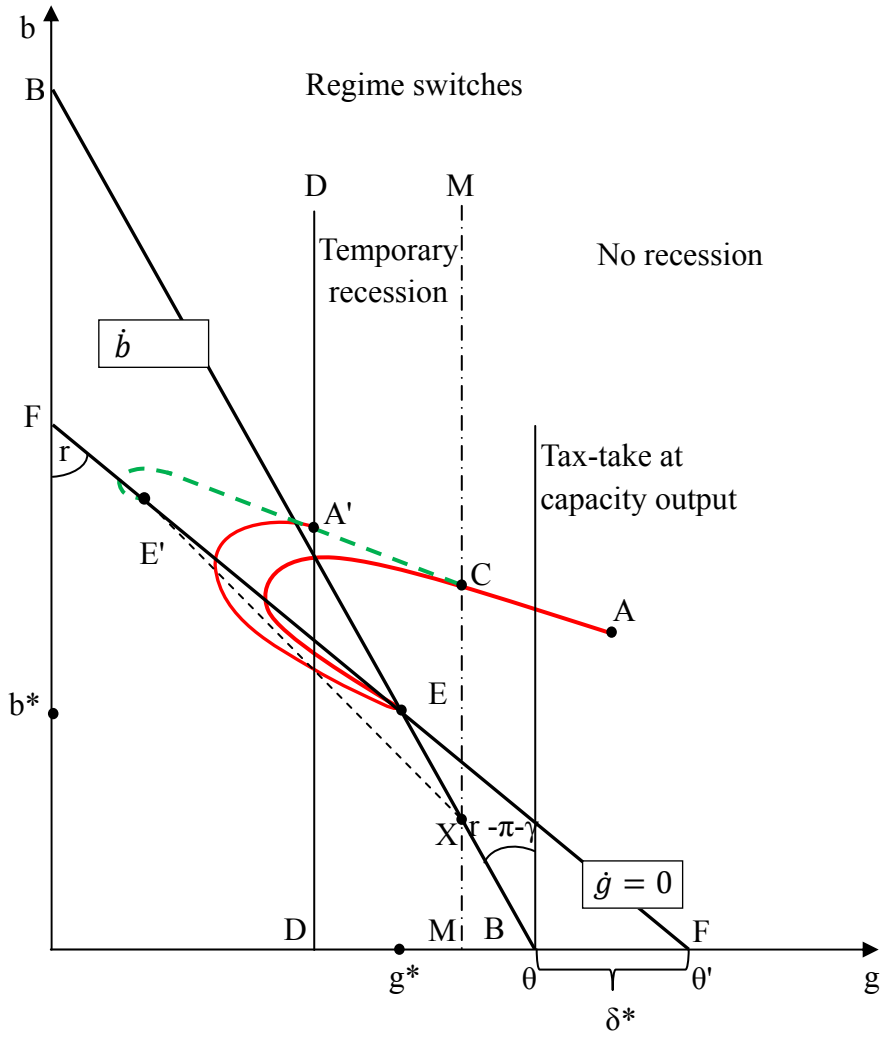

Figure 7. Fiscal consolidation – waiting and hoping.

How the economy might evolve in the period of "waiting and hoping" for private sector demand to replace government spending and get output back to capacity is indicated by the simulations in Figure 8, where the axes are the same but the variables are measured from the equilibrium at E. The bottom, hooked-shaped trajectory shows the phase path of the baseline model, while the middle line shows the equivalent when taxes are endogenous (and it is assumed that recession begins to the right of θ). For convenience, the ratio of debt to actual GDP is also plotted in the Figure and appears as the uppermost line, rising sharply as spending is cut.

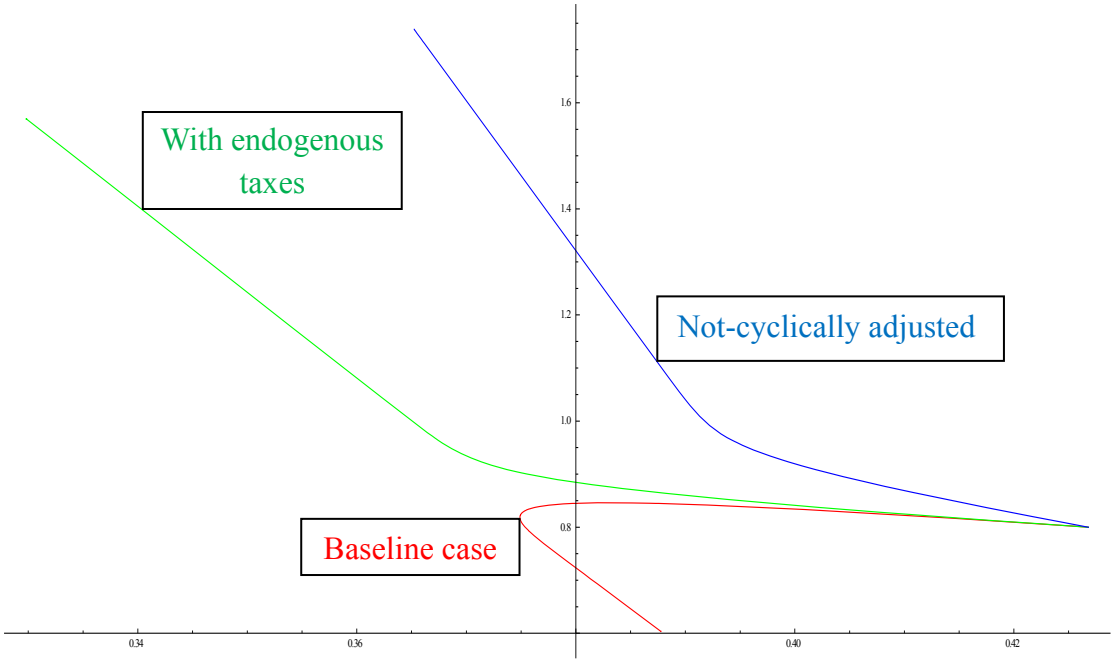

Figure 8. Simulations during the period of waiting and hoping.

What happens when demand recovers is shown by the schedule DD in Figure 7, indicating the end of recession. When the trajectory hits DD at A′, the dynamic path will thereafter be as in the baseline: there will be a regime switch with the economy now heading towards E instead of E′ as shown in the Figure.

Even though the economy will ultimately reach point E, we find that, even with a multiplier as low as 1, substantial losses of output may be involved as the stabilising effect of government spending is removed for a period of say five years. Starting from initial values of spending of 42.7% of GDP (10% above equilibrium g^*) and debt of 80% of GDP, for example, with a parameter of α= 0.6 government spending falls to 37.5% after 5 years, much as in the baseline model. But debt relative to capacity rises above 90% with endogenous taxes (as opposed to 84% in the baseline model), with the debt to actual GDP rising above 96% as output falls. The accumulated loss of output relative to capacity amounts to some 19 percentage points in the first 5 years.

3.3 Depression economics: the consolidation trap

In both cases analysed above, there has been no alteration in the fiscal plan despite the emergence of recession. But what if the evolution of the economy leads the authorities to change the plan? In their paper on "The challenge of debt reduction with fiscal

consolidation", Eyraud and Weber (2013) warn of what might happen using the ratio of debt to GDP as an operational fiscal target:

> If country authorities focus on the short-term behavior of the debt ratio, they may engage in repeated rounds of fiscal tightening in an effort to get the debt ratio to converge to the official target, undermining confidence, and setting off a vicious cycle of slow growth, deflation and further tightening. A possible solution could be to monitor debt ratios and set targets in cyclically-adjusted terms (p.21).

Even though we focus on cyclically-adjusted debt targets in our analysis, there is, it seems, a risk of getting caught in a consolidation trap. If the fall of income and taxes during consolidation is such that the target debt to capacity ratio looks like it will be missed, markets may lose confidence in the existing policy and authorities may be tempted to tighten fiscal policy, heading to an equilibrium with substantial unused capacity as long as demand remains weak.

Formally, the tightening of fiscal policy in the consolidation trap - and the recession this may cause - is determined as follows:

> Proposition 3: In the endogenous tax model with $g^*_{base} < \hat{g}_0$, if the target for debt it the same as in the baseline, $b^*_{En} = b^*_{Base}$, then the target of the fiscal consolidation needs to adjust to $\delta^*_{En} = \delta^*_{Base} + \varphi(g^*_{Base} - \hat{g}_0)/(1-\varphi) < \delta^*_{Base}$. In this case, the steady state spending is $g^*_{En} = (g^*_{Base} - \varphi\hat{g}_0)/(1-\varphi) < g^*_{Base}$; and there will be steady state recession: $y^*_{En} = 1 - \mu(\hat{g}_0 - g^*_{Base})/(1-\varphi) < 1$.

The adjustment to the fiscal plan described in Proposition 3 can be seen in Figure 9 where the target deficit, δ*, is reduced so as to ensure that the debt target of b^* is achieved despite the shift of BB, as discussed above.

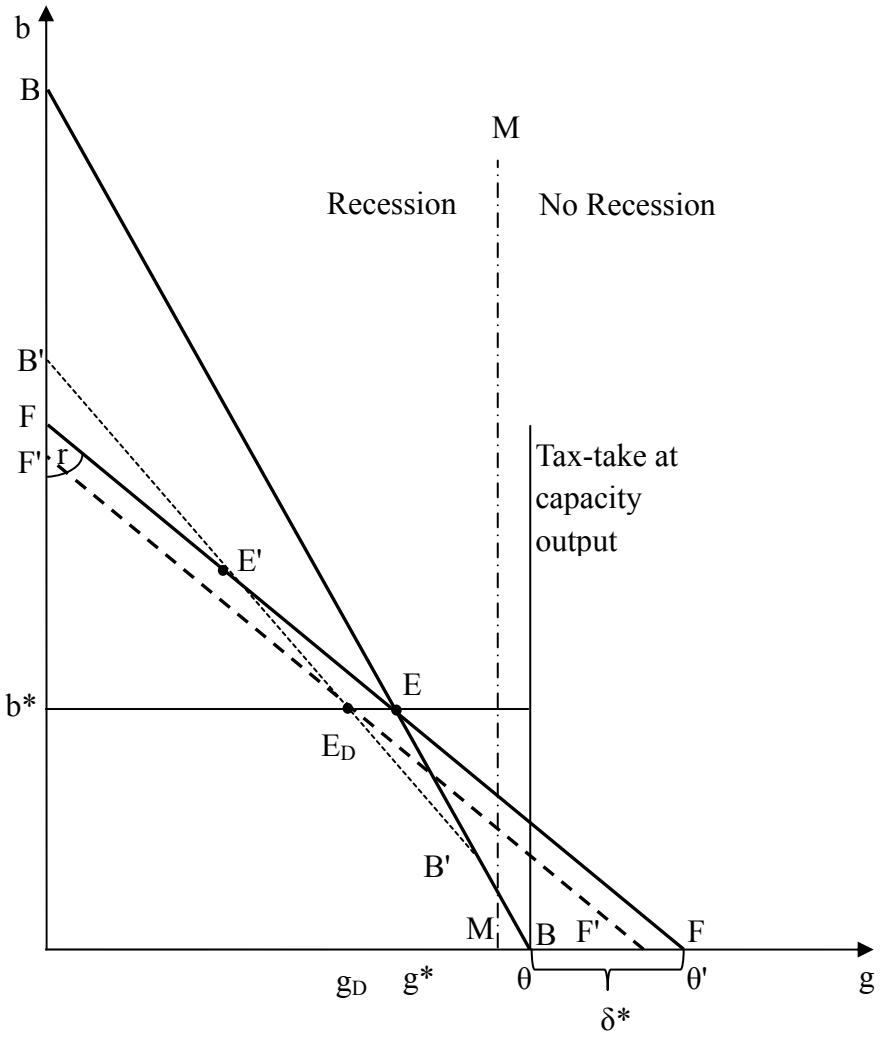

Figure 9. Tightening fiscal policy to hit the debt target, b*

The equilibrium labelled E' is where things were heading before private sector demand recovers; but with the tightening of policy, equilibrium shifts to E_D. To hit the debt target requires a greater tightening of fiscal policy because of the contraction of the tax base: but this drives the level of economic activity even lower. Until and unless there is a recovery of private demand, the economy will be heading into prolonged recession.

To gauge the effect of tightening the structural deficit, we use the same parameter values as in Section 3.2 to generate dynamics of debt and spending. With a multiplier of 1 and a targeted long-run debt to GDP ratio of 60%, we start with debt at 80% of GDP and an initial value of spending equal to $\hat{g}_0$ of 42.7%, which is 10% above g^*_{Base}= 38.8% and a structural deficit of 7.5%. To ensure the 60% long-run debt to GDP target, the fiscal target has to be shifted from a deficit of 1.2% to a surplus of

1.4%, which implies a tightening of the target for government expenditure of about 2.6% of capacity output. The phase diagram for debt and spending after such a tightening is shown in Figure 10, with the baseline model shown by the middle schedule and the consolidation trap shown by the left hand schedule; the right hand schedule shows the values relative to current GDP.

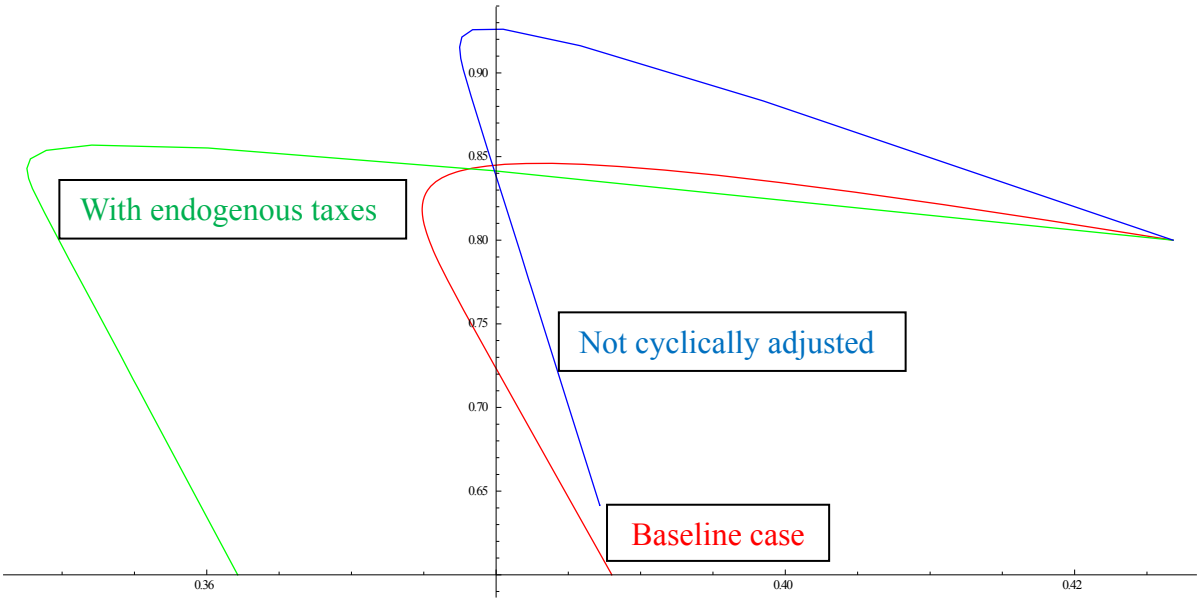

Figure 10. Simulations showing the effect of the tightening of structural deficits.

While the debt dynamics are similar to the baseline case, output is being driven towards a steady state level 6.5% lower than capacity as result of the 'consolidation trap'. The accumulated loss of output relative to capacity now amounts to some 27 percentage points in the first 5 years. Falling into the consolidation trap, by tightening fiscal policy so as to hit the debt target even in depression, could over 5 years increase output losses by 8% of capacity GDP, but the effect will be smaller if it takes time to fall into the trap.

As an illustration of the risks involved here, consider the UK case. Looking forward from 2011 on optimistic assumptions regarding demand recovery, it was forecast that the public sector net debt to GDP would rise from 53% to 69% of GDP over the next 5 years (OBR, 2011). After two years of almost no growth, however, the forecast for 2015-16 had increased to 85% (OBR, 2013), i.e. by 16 percentage points of GDP. Fortunately, the UK Chancellor of the Exchequer has preferred to allow slippage in consolidation rather than falling in to the trap: the 2015-16 target for the cyclically-adjusted surplus on the current budget – the government's "fiscal mandate" has been

reduced by 2 percentage points of GDP (from a surplus from 0.8% to a deficit of 1.2%).

4. Policy failure due to high interest rates

Though consolidation may only be achieved at substantial costs to output, the dynamics of the process are essentially stable. But this is not true if interest rates are high enough to violate the stability condition that $\varphi r = \theta \mu r < \gamma + \pi$. In that case, the BB schedule becomes flatter than FF and the pattern of generalized convergence under fiscal consolidation is lost - to be replaced by saddle point dynamics instead. So, even with output at capacity, fiscal consolidation will fail to hit its targets, typically diverging as shown by the trajectories in Figure 11.

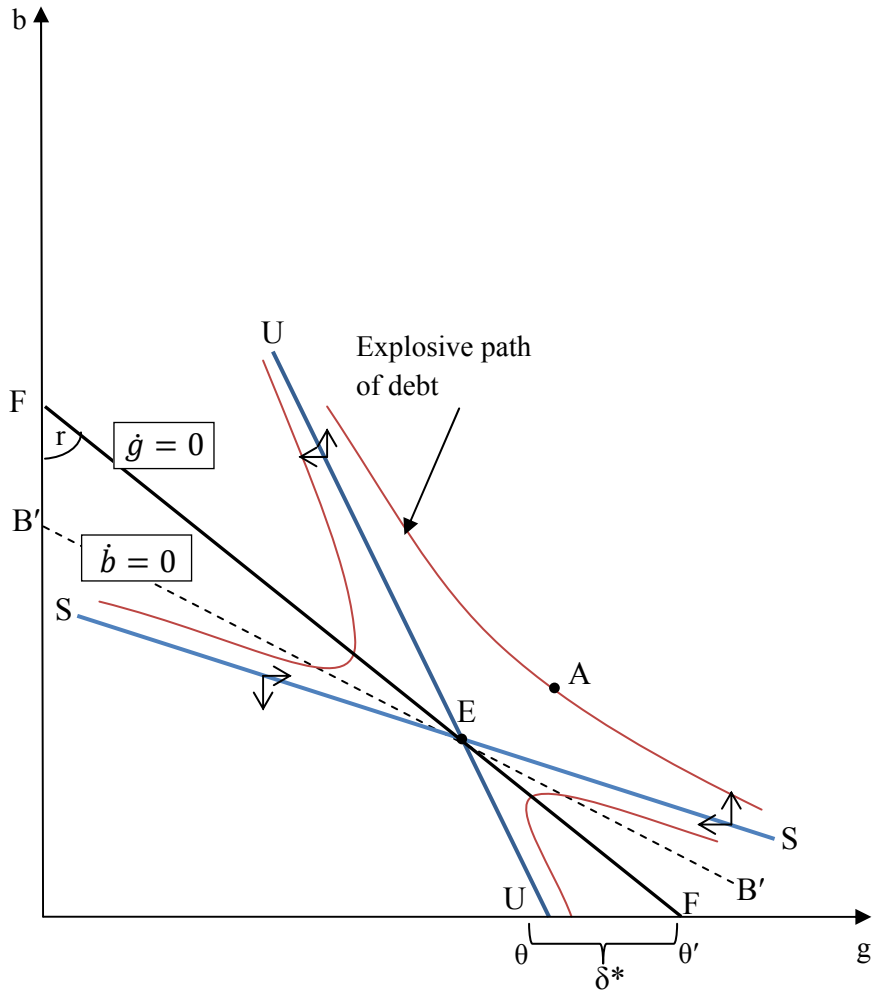

Figure 11. Fiscal consolidation defeated by high interest rates.

Unless the starting value happens to lie on the stable eigenvector, SS, sloping

downwards through E, it will never reach this equilibrium. From a starting point such as A for example, with high debt and high government spending, the trajectory will proceed continuously to the north-west, asymptotically approaching the unstable eigenvector labelled UU, with debt exploding as public expenditure implodes.

4.1 Checking bond market crises: the "Draghi put"

In a classic paper, Calvo (1988) argued that expectations of sovereign default may be self-fulfilling. Might the same not apply in this context? As the stability condition $\varphi r < \gamma + \pi$ indicates, instability will result from high interest rates –as when panic affects market sentiment and sovereign spreads increase due to fears of default.

To avoid a self-fulfilling solvency crisis in a Eurozone economy afflicted by such panic, De Grauwe (2011) recommended official intervention by the ECB to put a ceiling on interest rates by buying sovereign debt in the secondary market. The programme of Outright Monetary Transactions (OMT), announced in 2012, did in fact promise such intervention, subject to conditionality to ensure fiscal sustainability. So far the announcement itself has succeeded in substantially reducing sovereign spreads for Italy and Spain without either country applying for the facility. Whether the ECB pledge to save the Euro is constitutionally acceptable in Germany is now a matter for legal debate, but the policy has been stoutly defended by Mr Draghi who argued "frankly when you look at the data, it's really very hard not to state that OMT has been probably the most successful monetary policy measure undertaken in recent times…Ten-year sovereign bond yields declined spectacularly in several countries but went up in Germany" (Steen, 2013).

4.2 A desperate remedy: consolidation come what may

Even when the stability conditions for orthodox fiscal consolidation fail to be satisfied, there is, as we have seen, one path that leads to the desired equilibrium debt/capacity ratio. Could fiscal sustainability not be achieved by forcing the system onto this saddle path – by write downs or debt exchanges for example? That is the desperate remedy we examine here.

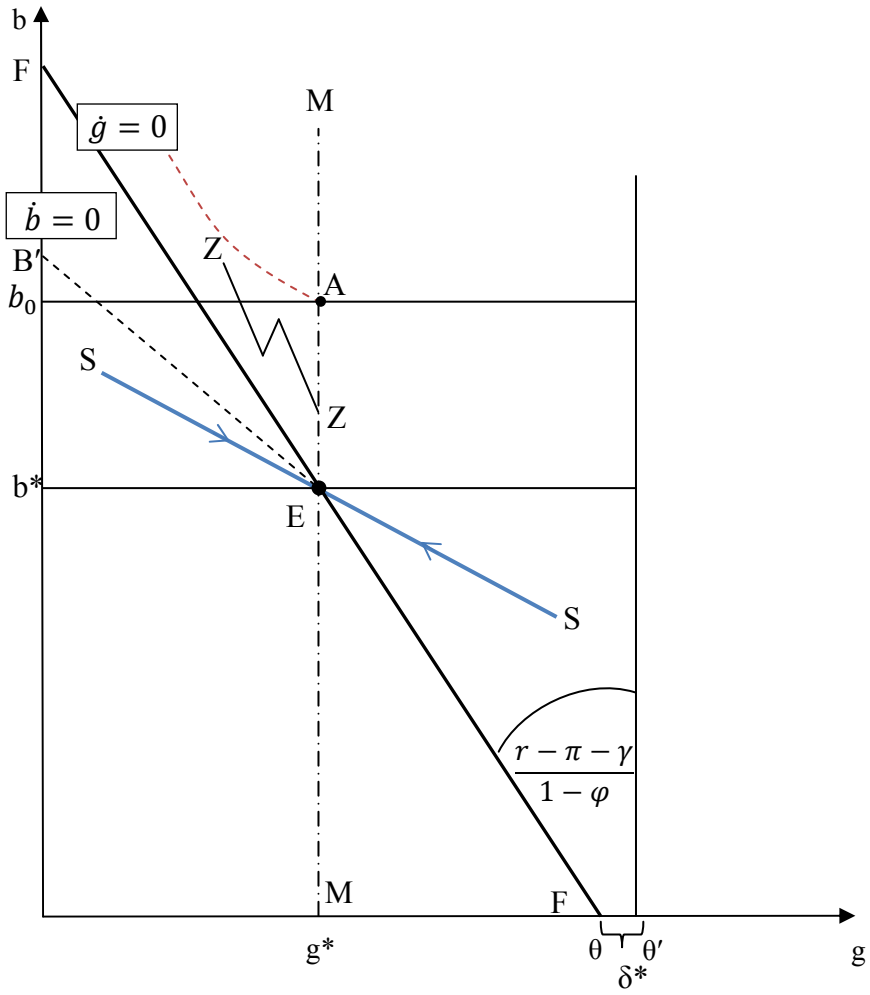

Figure 12. Failed attempts to stabilise.

Take an economy with good growth prospects but with high sovereign debt financed at exorbitant real rates of interest. Assume that the level of expenditure needed to maintain capacity is the same as the fiscal target, i.e. $\hat{g}_0 = g^*$. Given the starting value at point A in Figure 12, for example, consider what might be involved in trying to get to equilibrium at E.

By construction, a debt write-down would be sufficient to take the economy from A to E if sufficiently deep. In a widely-cited critique of IFI policy towards HIPC countries, however, Jeffrey Sachs (2002) argued that official debt relief on offer was, in general, insufficient to allow for successful exit from debt traps. Could the same be true of IMF approved policies of fiscal consolidation for non-HIPC countries too?

What if a series of write-downs were to be implemented over time, each insufficient to put the economy on the stable path? By assumption, sustainability will not be

achieved: the fiscal outcomes will instead follow a zigzag path such as that indicated by ZZ in the Figure, with rising debt and falling output. Note also that, since the system will now lie to the left of MM, there will be recession with its attendant multiplier effects, so the schedule EB′ in Figure 12 will be flatter than in Figure 11.

The assumptions made are probably too optimistic. There could, for example, be a deficiency of aggregate demand at the time of policy implementation, as discussed in section 3; and the failure of consolidation to hit its targets could then lead to a tightening of fiscal targets as discussed by Eyraud and Weber (2013) - see Proposition 4 in Appendix C for the required adjustment.

The details of possible disequilibrium developments may be left for discussion elsewhere; but action along different lines is surely required. To restore stability, interest rates must be reduced below exorbitant levels. If, for example[7], there is instability because $r = 0.10, \pi = 0\, \gamma = 0.03,, \theta = 0.4, b^* = 100, \mu = 0.8, \varphi = \theta\mu = 0.32$ so $\varphi r > \gamma + \pi$, then a reduction of real interest rates from 10% to 7%, would satisfy the stability condition. Thus sustainability could, in principle, be achieved without any write-down.

Why might interest rate be so high? If they are rates on debts in domestic currency, it could be that the banks are simply collecting seigniorage from the tax payer, so the requisite action will be to reduce the monopoly profits of banking. This was apparently one of the actions taken in Turkey to regain debt sustainability (see Kaya and Yilar, 2011).

5. Two Policy Alternatives

5.1 Keynesian stabilisation

If the failure of aggregate demand is temporary, there is an obvious case for postponing consolidation until demand recovers. In the interim, indeed, the over-riding priority is to stabilise the economy rather than the deficit: this is the perspective of DeLong and Summers (2012) who, for good measure, assume that potential output itself is endogenous and will be reduced as a result of slack. (They suggest a reduction

[7] For simplicity this example ignores the inflation typical of such cases.

of about one-tenth of the accumulated slack prior to recovery[8].)

If consolidation is postponed till recovery takes place at some time T, and fiscal spending continues at a high level to maintain output at potential in the period of demand failure, this will lead to a rise in the debt output ratio, as shown by the arrow from A to D in Figure 13. Note that, by definition, this lies on the MM schedule as $g = \hat{g}_0$ as public expenditure is being used as a prop to maintain aggregate demand. Debt will increase but the recession will be avoided.

For comparison, we include the implication of proceeding immediately with fiscal consolidation, as shown by the trajectory from A to C, which lies to the left of MM, as prompt consolidation causes recession. How far this succeeds in reducing the debt to capacity level by time T relative to the policy of economic stabilisation will depend on the parameters: but the loss of income and of associated taxes hardly commends it as an efficient alternative.

Given recovery of private sector demand at time T, fiscal consolidation at capacity levels of output will now be required in both cases, starting at points D and C respectively – where capacity may be impaired at point C if there are hysteresis effects. It should be noted that DeLong and Summers propose that the extra debt incurred on account of economic stabilisation be added to the target stock of debt (taking it from b^* to b^{**} in the figure), with the extra cost of debt service covered by extra taxation shifting to the right the line of stationarity for g (see the dashed line) at some cost to supply potential. The additional fiscal consolidation needed when recovery has taken place would thus take debt and expenditure from D to E□.

[8] So if, for example, Europe were to have suffered a cumulative slack of 10% due to the effects of fiscal tightening for two years as the simulations of the NIESR by Holland and Portes (2012) appear to show, there would be a 1% reduction in the level of economic potential thereafter as a result.

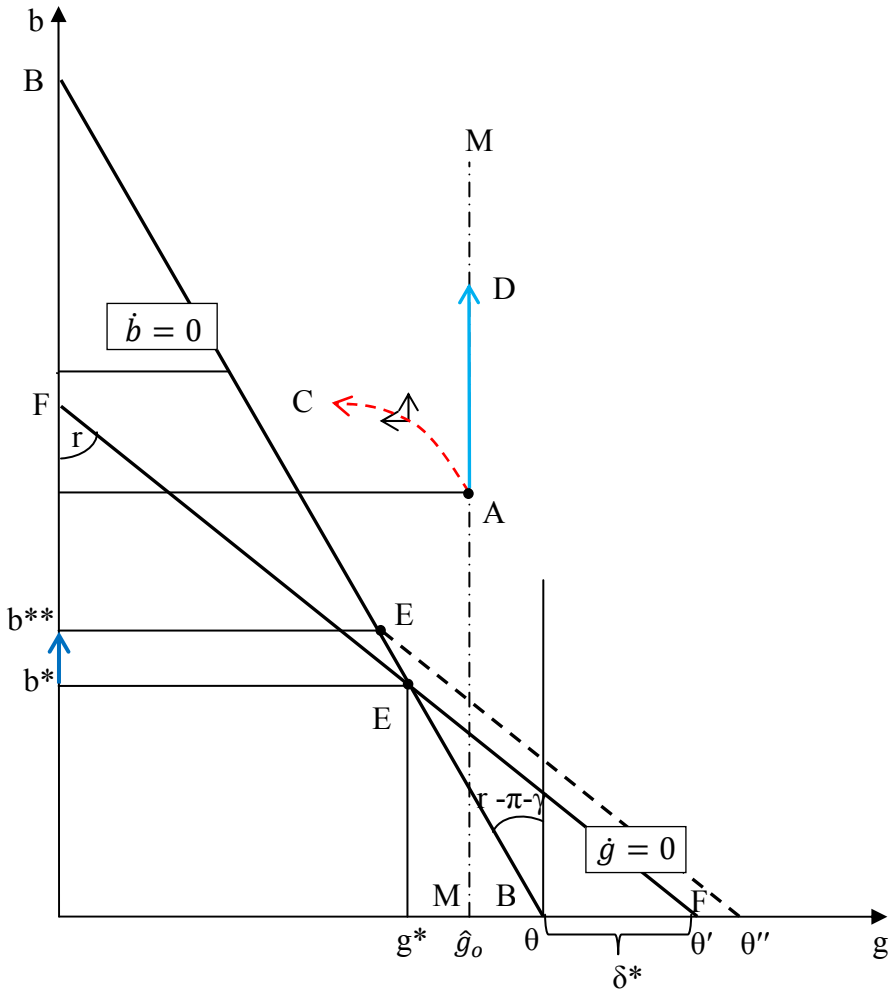

Figure 13. DeLong and Summers: stabilisation delays fiscal consolidation.

DeLong and Summers argue that, the welfare costs of the tax increase needed to cover the interest on the extra debt are far smaller than the benefits, for the latter will include both avoiding recession - and the negative effects of hysteresis over the indefinite future[9].

Those who support fiscal tightening in depression appeal to time consistency arguments - if a government does not tighten now it never will; and to arguments of equity – that debt represents an intergeneration transfer, see Williamson (2013). These are not issues we discuss here, but it would surely be better to find more efficient commitment devices by institutional mechanisms and seeking intergenerational equity via taxation rather than by running the economy below capacity.

[9] For a calibration of their argument for the UK case see Miller and Roberts (2013, forthcoming).

5.2 Indexing public debt to GDP

In Barr et al. (2012) the case is made for state contingent debt instruments so that a shock to GNP will reduce debt payments in line with a reduction in income.

> The debt's redemption value is linked to the level of GDP, which means that if a sovereign issues only GDP-linked bonds, its entire debt stock will adjust in proportion to GDP. The interest rate on the bond is defined as a fixed percentage of this principal, so it too adjusts with GDP…Shocks to GDP growth no longer enter into this debt dynamics equation. (Barr et al. 2012 p.17)

With such debt instruments in place, the fall in GDP should not change the debt-income ratio, so the path for consolidation in Figure 4 would begin at E and not A. Without considering the details of adjustment it's easy to see how such instruments could be useful in avoiding the dangers of fiscal fatigue.

> One of the major benefits of GDP linked bonds identified by Borensztein and Mauro (2004) is that they can reduce the need for governments to undertake pro-cyclical fiscal policy during recessions…GDP-linked bonds can break this 'vicious circle' because sovereigns do not need to undertake fiscal consolidation to stabilise debt. Instead, debt is stabilised because the face value of GDP-linked bonds is reduced. (Barr et al. 2012 p.23)

A similar case for GDP bonds is made by Griffith-Jones and Hertova (2012, p.6-8):

> First, they stabilize government spending and limit the pro-cyclicality of fiscal pressures [in borrowing countries] by requiring smaller interest payments at times of slower growth—providing space for higher spending or lower taxes—and vice versa…Second, by allowing debt service ratios to fall in times of slow or negative growth[10], GDP-indexed bonds reduce the likelihood of defaults and debt crises. …[Investors, too] would benefit from a lower frequency of defaults and financial crises, which often result in costly litigation and renegotiation and sometimes in outright large losses…[and] the bonds would provide an opportunity for investors to take a position on countries' future growth prospects, offering investors equity like exposure to a country.

5.3 An SPV to assist the swap.

Chapter 11 procedures suggest that financing problems for debtor countries can be eased by bond swaps. There is also the problem of funding instability as creditors switch between countries in a flight for safety. Creditor panic in bond markets with a

[10] If half of Mexico's total government debt had consisted of GDP-indexed bonds, it would have saved about 1.6 percent of GDP in interest payments during the financial crisis in 1994/1995 (Borenzstein and Mauro 2004).

flight to quality which has taken bond yield in Germany to virtually zero suggests the need for some consolidation. A number of plans have been proposed, as indicated in Table 1, from Leinemann (2012). .

Name	Concept
Euro-bonds	Issue of common bonds to replace all debt
"Blue bonds"	Issue of common bonds up to 60% of GDP
"Elite" bonds	Common bonds only for AAA rated countries
Debt retirement fund	New entity that pools all debts above 60% of GDP, issues its own common bonds. Countries have a credible commitment to amortise the debt in a certain time frame

Table 1. Different types of stability bonds.

What we propose however is a European SPV to play the role of market-maker (in taking up state-contingent debt issued in swaps) and to act as market-stabiliser (by offering 'supra-national' Eurobonds to investors). This is shown graphically as follows:

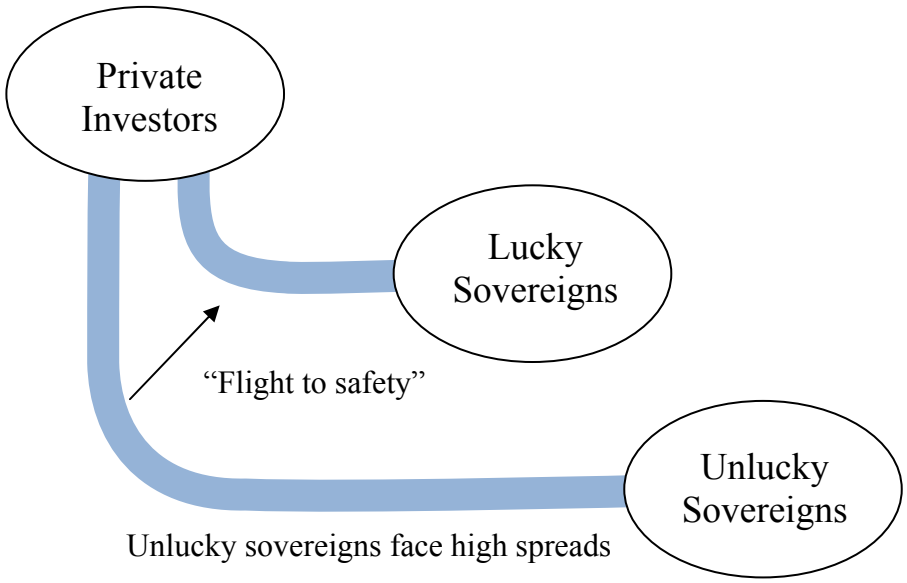

Figure 14. BEFORE: Investors holds sovereign bonds - but are prone to switch

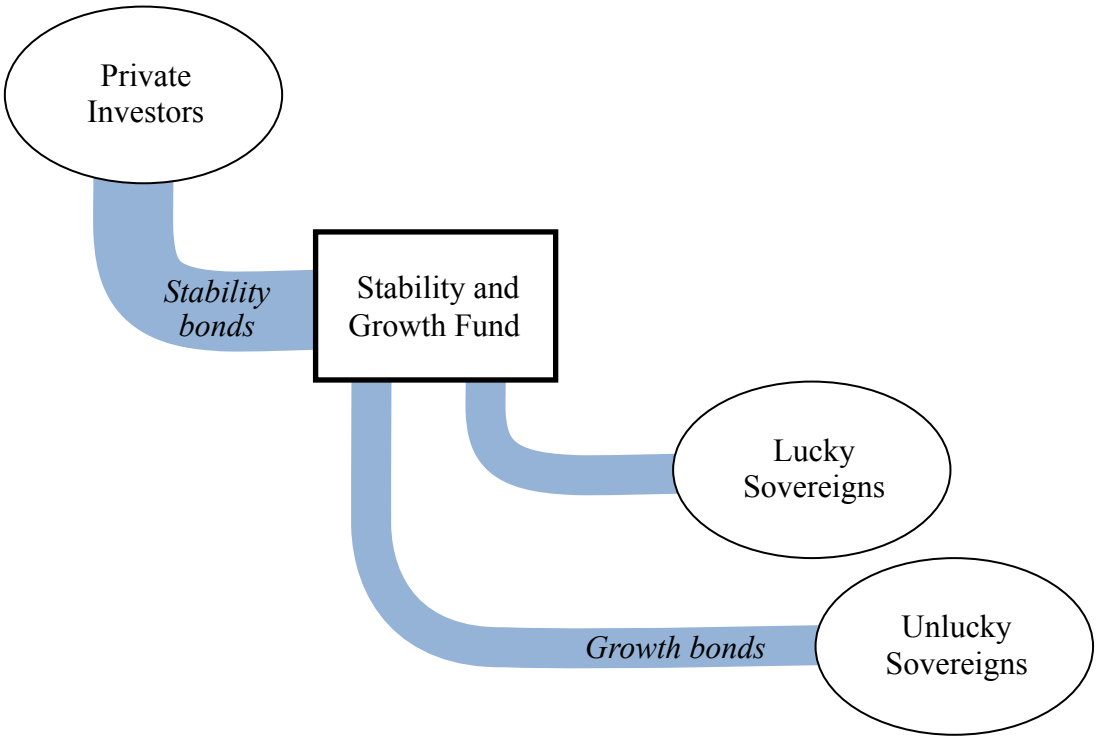

Figure 15. AFTER: Stability and growth fund pools sovereign debt - and diversifies types of bond

In more detail, consider the balance sheet of the proposed SPV.

Assets	Liabilities
Sovereign bonds: (a) Plain vanilla (b) Growth and GDP-linked	Euro stability bonds Equity base

Table 2. Balance sheet of SPV

On the assets side, holding of regular government debt are complemented by growth and GDP-linked bonds. The market may not give full value for them especially if they are issued in the midst of a crisis. This was the case for Argentina; as reported by Griffith-Jones and Hertova (2012) growth warrants now standing at about $15 were sold for only $2 in 2005: and it appears that the market attached little value to Greek warrants. This suggests that such securities might be taken out of the market by an agency with a longer horizon - until such time as the country has begun to grow. This would give a breathing space for the debtor country and help it to avoid selling its debt at a deep discount. That is the logic behind the role of a European SPV for the purpose. Its liabilities will be the Eurobonds; and its equity will be guaranteed by

Treasuries of Europe.

The issue of debtor moral hazard is flagged up by Reinhart and Rogoff (2009, xli-xlii) as follows: "Suppose a world government agency provided expansive deposit insurance to protect every worthy borrower from panics…The problem is that if one provides insurance to everyone everywhere, with no conditions, some players are going to misbehave." But what is envisaged is not blanket insurance for all European debt: it is tailored restructuring subject to strict conditionality. As GDP is observable and verifiable, the first best contractual solution would be to use GDP in designing contingent instruments. Even if debt service capacity is not monotonically related to GDP, "Eurozone conditionality" can surely be used to link debt-service capacity to GDP so as to reduce moral hazard.

6. Conclusions

Reinhart and Rogoff (2010) have warned that public sector debt levels in excess of 90% of output will be associated with low or negative economic growth. For advanced economies since World War II, they warn specifically that: "real GDP growth is relatively stable around 3 to 4 percent until the ratio of public debt to GDP reaches 90 percent. At that point and beyond, average GDP growth drops sharply to zero or slightly negative." In this paper we discuss how such warnings can become a self-fulfilling prophecy, by inducing rapid fiscal consolidation at a time of a demand-deficient recession.

Martin Wolf (2013b) observes:

> What looked, until mid-2010, to be a burgeoning recovery from the nightmare of the "Great Recession" was aborted, notably so in the UK and eurozone.

and goes on to conclude that:

> the time for rapid structural fiscal tightening comes only after the private sector starts to eliminate its structural financial surpluses. That would not be so soon after the crisis. It would also require prior restructuring of the financial sector and writedowns of excessive private debt.

This is the first policy alternative we have discussed: putting economic stabilisation before fiscal consolidation; the second is the issue of state-contingent debt.

Does the experience of no growth and rising debt in Europe provide convincing

evidence of self-defeating austerity? Does the failure to hit ambitious fiscal targets indicate a shift in the direction of policy towards a Keynesian perspective – or will it trigger great fiscal tightening as policy falls into the consolidation trap? Is there evidence of the hysteresis effects that DeLong and Summers emphasise? Can governments be trusted to consolidate outside times of crisis? These are all ascinating questions, but answering them would take one far beyond the simple analytics presented in this paper.

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Appendix

Appendix A. Debt-spending dynamics in the baseline model

Let $\gamma' = \gamma + \pi$**.** The linear system in (3) has the following general solutions

$$\begin{bmatrix} b \\ g \end{bmatrix} = A_1 \begin{pmatrix} 1 \\ r - \gamma' - \lambda_1 \end{pmatrix} e^{\lambda_1 t} + A_2 \begin{pmatrix} 1 \\ r - \gamma' - \lambda_2 \end{pmatrix} e^{\lambda_2 t} + \begin{pmatrix} b^* \\ g^* \end{pmatrix} \quad \text{(A.1)}$$

where A_1 and A_2 are two arbitrary constants to be determined by boundary conditions, the steady-state debt-spending is given by

$$\begin{pmatrix} b^* \\ g^* \end{pmatrix} = \begin{pmatrix} (\theta' - \theta)/\gamma' \\ \theta' - r(\theta' - \theta)/\gamma' \end{pmatrix} = \begin{pmatrix} \delta^*/\gamma' \\ \theta' - r\delta^*/\gamma' \end{pmatrix}. \quad \text{(A.2)}$$

and the eigenvalues are given by

$$\lambda_1 = \frac{-(\alpha - r + \gamma') + \sqrt{(\alpha - r + \gamma')^2 - 4\alpha\gamma'}}{2}$$

and

$$\lambda_2 = \frac{-(\alpha - r + \gamma') - \sqrt{(\alpha - r + \gamma')^2 - 4\alpha\gamma'}}{2}$$

It is clear that the system is stable and converges to its steady state if $\alpha - r + \gamma' > 0$, and exhibits cycles when $0 < \alpha - r + \gamma' < 2\sqrt{\alpha\gamma'}$. We focus on the case of convergence without cycles where $\alpha - r + \gamma' \geq 2\sqrt{\alpha\gamma'}$.

Note that the stability of the solution to (3) also depends on a natural boundary condition that

$g \geq 0$. If the debt level for g=0 is $b \geq \theta/(r - \gamma')$, debt is not sustainable. Otherwise, it is.

This also applies to the case of fiscal fatigue: if the debt level when spending reaches its lower bound is too large, debt is not sustainable; otherwise it is.

Appendix B. Debt-spending dynamics when taxes are endogenous

For modified system given by (3'), the general solutions are

$$\begin{bmatrix} b \\ g \end{bmatrix} = A_1' \begin{pmatrix} 1-\varphi \\ r-\gamma'-\lambda_1' \end{pmatrix} e^{\lambda_1' t} + A_2' \begin{pmatrix} 1-\varphi \\ r-\gamma'-\lambda_2' \end{pmatrix} e^{\lambda_2' t} + \frac{1}{\varphi r-\gamma'} \begin{pmatrix} \varphi(\theta'-\hat{g}_0)-\delta^* \\ r\delta^*-\gamma'\theta'+\varphi r\hat{g}_0 \end{pmatrix}$$

(B.1)

where A_1' and A_2' are two arbitrary constants to be determined by boundary conditions, the eigenvalues are given by

$$\lambda_1' = \frac{-(\alpha - r + \gamma') + \sqrt{(\alpha - r + \gamma')^2 - 4\alpha(\gamma' - r\varphi)}}{2}$$

and

$$\lambda_2' = \frac{-(\alpha - r + \gamma') - \sqrt{(\alpha - r + \gamma')^2 - 4\alpha(\gamma' - r\varphi)}}{2}$$

It is clear from the eigen-values that the stability conditions are $\alpha - r + \gamma' > 0$ and $\gamma' > r\varphi$, but the condition for cycles becomes $0 < \alpha - r + \gamma' < 2\sqrt{\alpha(\gamma' - r\phi)}$. How the introduction of the endogenous taxes changes the shapes of manifolds is discussed in the text.

Let b_{Base}^* and g_{Base}^* denote steady state debt and spending in the baseline model, and b_{En}^* and g_{En}^* represent steady state debt and spending in the endogenous tax model, then from (A.2) and (B.1):

$$b_{Base}^* = \delta^*/\gamma' \tag{B.2}$$

$$g_{Base}^* = \theta' - r\delta^*/\gamma' \tag{B.3}$$

$$b_{En}^* = \frac{\varphi(\theta'-\hat{g}_0)-\delta^*}{\varphi r-\gamma'} \tag{B.4}$$

$$g_{En}^* = \frac{r\delta^*-\gamma\theta'+\varphi r\hat{g}_0}{\varphi r-\gamma'} \tag{B.5}$$

When $\hat{g}_0 = g_{Base}^*$, substitution of (B.3) into (B.4) and (B.5) yields

$$b_{En}^* = \frac{\delta^*}{\gamma'} = b_{Base}^*$$

and

$$g_{En}^* = \theta' - \frac{r\delta^*}{\gamma'} = g_{Base}^*.$$

From (9), one can derive the steady state income as

$y_{En}^* = 1 - \mu(\hat{g}_0 - g_{En}^*) = 1 - \mu(\hat{g}_0 - g_{Base}^*) = 1.$

So as long as $\hat{g}_0 = g_{Base}^*$, steady state values of debt, spending and output are the same as those in the baseline model.

When $\hat{g}_0 > g_{Base}^*$, imposing

$b_{En}^* = b_{Base}^*$

results in

$\delta_{En}^* = \delta_{Base}^* - \varphi(\hat{g}_0 - g_{Base}^*)/(1 - \varphi)$ (B.6)

where δ_{Base}^* and δ_{En}^* represent targets of fiscal consolidation in the baseline and endogenous tax models respectively.

Substitution of (B.6) into (B.5) yields the steady state level of spending in the endogenous tax model:

$g_{En}^* = (g_{Base}^* - \varphi\hat{g}_0)/(1 - \varphi).$ (B.7)

Substitution of (B.7) into (9) yields the steady state income in the endogenous tax model

$y_{En}^* = 1 - \mu(\hat{g}_0 - g_{En}^*) = 1 - \frac{\mu(\hat{g}_0 - g_{Base}^*)}{1-\varphi} < 1.$

Appendix C. Endogenous tax model with instability

Note that if one of the stability conditions in Appendix B is violated, the system (B.1) is saddle stable, with $\lambda_2' < 0 < \lambda_1'$. The steady state values for different $\hat{g}_0$ follows directly from the proof in Appendix B.

Proposition 4: If the system is saddle stable, the size of the consolidation trap is determined as follows:

1) If $\hat{g}_0 > g_{Base}^*$ and $b_{En}^* = b_{Base}^*$, then the target of the fiscal consolidation needs to adjust to $\delta_{En}^* = \delta_{Base}^* + \varphi(g_{Base}^* - \hat{g}_0)/(1 - \varphi) < \delta_{Base}^*$. In this case, the steady state spending is $g_{En}^* = (g_{Base}^* - \varphi\hat{g}_0)/(1 - \varphi) < g_{Base}^*$; and there will be steady state recession: $y_{En}^* = 1 - \mu(\hat{g}_0 - g_{Base}^*)/(1 - \varphi) < 1$.