Exchange Rate Crises and Fiscal Solvency

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Abstract

This paper combines insights from generation-one currency crisis models and the Fiscal Theory of the Price Level (FTPL) to create a new generation-one type model. Fiscal solvency is the fundamental generating crises, as in generation-one models. The initial fixed-exchange-rate policy entails risks, both to its sustainability and to the real value of government debt. The risks are due to stochastic surplus shocks and an upper bound on the present value of surpluses. Stochastic surplus shocks, changes in expectations of future fiscal commitments, and changes in the policy parameters can raise current desired debt or reduce expected future surpluses. Should the government's desired debt exceed the present-value of expected future surpluses, agents refuse to lend into this position of insolvency. The sudden stop of capital inflows creates a crisis. Equilibrium can be restored with some combination of policy switching and debt devaluation to restore fiscal solvency. The model can explain a wider variety of crises than generation one models, including those involving sovereign default. It is applied to explain crises in Argentina (2001), Mexico (1994-95), and East Asia (1997), which did not fit the stylized facts of generation one models.

Key Words: Currency Crises, Generation One Currency Crisis Models, Fiscal Theory of the Price Level, Policy Switching, Passive Fiscal Policy, Active Fiscal Policy, Sovereign Default

JEL Codes: F31, F33, E42, E44, E63

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1 Introduction

The generation-one model of exchange rate crises (Krugman 1979, Flood and Garber 1984) provided invaluable insights into the causes of exchange rate crises, offering an explanation for many of the crises of the 1980's, in which government budget deficits and declining reserves played prominent roles. Yet, the model failed to explain many crises after 1990, including those with obvious fiscal roots (Argentina 2001), as well as those with less obvious fiscal roots (Mexico, 1994-95; East Asia 1997). The model failed to explain the Argentine crisis because the central role for reserves implies that a currency-board country should have been immune from a speculative attack. It failed to explain crises in Mexico and East Asia because these countries did not have government budget deficits, and because Mexico sterilized the effects of the speculative attack on its money supply.

These issues led researchers to modify the generation-one model with specifics which would allow it to explain particular crises, and to develop new generations of exchange rate crisis models. New-generation models do not use fiscal solvency as a fundamental determinant of crises. We argue that many exchange rate crises, including many of those which occurred in the 1990's, can be explained by a model which retains fiscal insolvency as the fundamental generating the crisis.

We combine insights from generation-one currency crisis models and the Fiscal Theory of the Price Level (FTPL) to create a new generation-one type exchange rate crisis model.

Fiscal solvency is the fundamental generating the crisis, as in generation-one models. The new model replaces the assumption that policy is unsustainable with the assumption that baseline fiscal policy is sustainable (Bohn 2008) and passive (Leeper 1991). We assume that the primary surplus is governed by a rule in which it responds positively to lagged debt. This implies that at the initial price level, the expected present value of debt in the limit is zero, assuring fiscal solvency. Active monetary policy fixes the exchange rate.

However, without modification, the baseline fiscal policy is just as unreasonable as the reserve-financed budget deficits in the original generation-one model. Policy-makers do respond to shocks in the economy, causing the primary surplus to deviate from the value determined by past debt. Such behavior is exemplified most recently by the policy-response to the global recession of 2008-2009. Generation-two models (Obstfeld 1994) explicitly account for such reactions with the assumption that governments optimally change policy (exchange rate depreciation) when a trigger value for a particular variable is reached. A reasonable specification of fiscal policy must allow policy-makers to deviate from the baseline adjustment of the primary surplus to debt. We model these deviations as stochastic shocks to the primary surplus. These shocks include fiscal reaction to the state of the economy as well as non-economic shocks due to politics or war.

Even with stochastic fiscal shocks, as long as the primary surplus responds positively to lagged debt, the expected present value of the debt is zero in the limit, assuring fiscal solvency. However, a series of negative shocks could require very large values for future primary surpluses to service the debt, and every government faces limits on its abilities to raise taxes. These limits imply an upper bound on the present value of future primary surpluses and, equivalently, on debt. When fiscal policy is subject to stochastic shocks and an upper bound on debt, there is risk that the initial policy mix might not be indefinitely viable. If a current or expected future shock creates the expectation that the government cannot service its desired debt at market interest rates, then agents refuse to lend. The sudden stop in lending prevents the government from borrowing to continue the initial fiscal policy and defines the crisis.

Since the crisis is caused by fiscal insolvency, its resolution requires a policy response to restore solvency and lending. In generation-one models, solvency is achieved with increased seigniorage revenues, created by increased money growth. The second major departure from generation-one models is that solvency can be restored through devaluation of nominal debt, created by currency depreciation. The model does not rule out an increase in seigniorage, but additional seigniorage is not necessary. Jeanne and Guscina (2006) document that a large fraction of emerging market debt is denominated in domestic currency in contrast to the "original sin" hypothesis. Burnside, Eichenbaum, and Rebelo (2006) present evidence that debt devaluation played a larger role than increased seigniorage in restoring fiscal solvency following many of the 1990's exchange rate crises.

We consider several types of policy responses. The first is a promise of fiscal reform, with a switch to active fiscal policy as in the literature on the "Fiscal Theory of the Price Level" (FTPL). With active fiscal policy, monetary policy must be passive, requiring exchange rate flexibility. The switch creates larger near-term primary surpluses, which serve to raise the expected present-value of primary surpluses. If the present-value of primary surpluses is still too low relative to debt after the policy switch, exchange rate depreciation, reducing the real value of outstanding government debt, is necessary to restore fiscal solvency. Since depreciation is possible, interest rates rise in anticipation of the crisis, raising the value of debt toward its upper bound and increasing the probability of a crisis. After the crisis, seigniorage does not have to rise, and if the monetary authority's inflation target does not change, sterilization of the effect of the speculative attack on the money supply is necessary. When the government responds with policy switching, the resulting model is a dynamic FTPL model of exchange rate crises.

The antecedents of the idea that passive fiscal policy, subject to stochastic shocks, might require increases in debt too large to be serviced by feasible primary surpluses are in Sims (1997). He outlines a stochastic policy-switching model with a switching hazard conditioned on outstanding government debt and shows that expectations of switching imply explosive debt due to increasing interest rates. The model developed here expands on Sims by specifying stochastic fiscal policy and allowing the initial policy to continue until no longer feasible, as in the original generation-one model, instead of collapsing with a hazard rate increasing in debt. Closed-economy policy-switching models include those of Davig and Leeper (2006) and Davig, Leeper, and Chung (2007), who consider the implications of exogenous policy-switching.

FTPL policy switching is not the only possible response to the sudden stop of capital flows. A government could devalue and repeg at a lower exchange rate, while maintaining the existing policy mix. We show, however, that such a policy implies a post-crisis period of instability with continued arbitrarily high interest rates and exchange rate depreciations. Alternatively, a government could receive an IMF loan to replace private capital flows, conditional on policy change which increases the present value of future primary surpluses, thereby restoring solvency. However, raising surpluses in the face of economic shocks which have reduced them could be politically and economically painful and might not be desirable when debt devaluation is available as a source of revenue. This could explain why IMF loans are often combined with exchange rate depreciation, allowing domestic-currency debt devaluation.

We use the model to explain several crises after 1990, which were arguably caused by fiscal shocks, but which the original generation-one model cannot explain. This demonstrates that fiscal solvency plays a greater role in causing exchange rate crises than the original generation one model would imply.

This paper is organized as follows. The next section presents the model. Section 3 characterizes dynamics in an FTPL policy-switching model of an exchange rate crisis. Section 4 considers other policy reactions, and Section 5 applies the model to several crises after 1990. Section 6 contains conclusions.

2 Model

2.1 Overview

In this section, we set up a simple model of a small open economy which we can use to address fiscal risk. The model contains four key assumptions. First, international creditors lend to a government only when they expect to receive the market rate of return. Second, the domestic government issues debt denominated in its own currency. Third, there is an upper bound on the value of government debt. Fourth, fiscal policy implies risk on government debt, reflecting the reality that a government's commitment to raise taxes to finance expenditures cannot be totally unconditional.

2.2 Goods and Asset Markets

We assume that there is a single good in the world, implying that goods markets equilibrium requires the law of one price. Normalizing the foreign price level at unity implies that the exchange rate, S_t, defined as the domestic-currency price of foreign currency, equals the domestic price level. We assume that the world interest rate (i) is constant. To keep the model simple, we set output growth to zero.[1]

The **first key assumption** is that international creditors are willing to buy the small economy's government bonds as long as its interest rate, i_t, satisfies interest rate parity. Interest rate parity can be derived, using the Euler equations for a representative world agent, when the covariance of the country's interest rate with world-agent consumption is zero, yielding

$$\frac{1}{1+i_t} = \left(\frac{1}{1+i}\right) E_t \frac{S_t}{S_{t+1}}, \tag{1}$$

where E_t denotes the expectation conditional on time t information.[2] The interest rate can

[1] The model is equivalent to one specified in terms of values as a fraction of GDP when the real interest rate is interpreted as the growth-adjusted real interest rate. We make this modification in the section where we apply the model to explain actual crises.

[2] Given nominal domestic and foreign-currency bonds $\left(B_{t-1},\ B^*_{t-1}\right)$, the normalization that the world price level is unity, and letting real foreign consumption and income be denoted by c^*_t and y^*_t, respectively, the world agent's real budget constraint is given by $c^*_t = (1+i)\, B^*_{t-1} + (1+i_t)\, \frac{B_{t-1}}{S_t} + y^*_t - B^*_t - \frac{B_t}{S_t}$. The Euler

rise above the world interest rate when there is some possibility of an exchange rate crisis which will be resolved with depreciation.[3]

2.3 Monetary Policy

Monetary policy is assumed to have a fixed exchange rate (price level) target.[4] When there is no possibility of a change in the exchange rate in the next period, interest rate parity from equation (1), implies that the domestic interest rate equals the world rate.

2.4 Fiscal Policy

2.4.1 Government Flow Budget Constraint

The **second key assumption** is that government bonds are denominated in domestic currency.[5] This assumption is based on work by Jeanne and Guscina (2006), who show that even in emerging markets, a substantial fraction of government debt is denominated in domestic currency. Additionally, Burnside et al. (2006) show that in several crises in the 1990's, debt devaluation was a larger source of government revenue than money growth.

Letting G_t and T_t denote nominal government spending and tax revenue, respectively, the government's nominal flow budget constraint is given by

$$B_t + M_t = (1 + i_{t-1}) B_{t-1} + M_{t-1} + G_t - T_t. \tag{2}$$

equations are $U'(c_t^*) = \beta E_t (1+i) U'\left(c_{t+1}^*\right)$ and $U'(c_t^*) = \beta E_t (1+i_t) \frac{S_t}{S_{t+1}} U'\left(c_{t+1}^*\right)$. The assumption that world consumption is uncorrelated with the domestic interest rate yields equation (1).

[3] When we consider the possibility of default, the interest rate parity equation must be modified such that expected returns on domestic debt, conditional on the possibility of default, equal world returns.

[4] We do not model the implementation of this target. It could be implemented in a model in which the monetary authority controls the nominal interest rate, controlling expected inflation. Control of the price level could then be achieved with an unstable policy rule, whereby the price level must jump to assure equilibrium.

[5] We could allow some government bonds to be denominated in foreign currency with no substantive change to the analysis, as long as some bonds are denominated in domestic currency. Magnitudes would change with larger depreciation needed the smaller the fraction of domestic-currency debt in total debt.

Defining real government debt (b_t) and the real primary surplus (s_t) as,

$$b_t = \frac{1}{P_t}\left[B_t + \frac{1}{1+i_t}M_t\right],$$

$$s_t = \frac{1}{P_t}\left[T_t + \frac{i_t}{1+i_t}M_t - G_t\right]$$

the government's flow budget constraint can be expressed as

$$b_t = (1+i_{t-1})\left(\frac{S_{t-1}}{S_t}\right)b_{t-1} - s_t. \tag{3}$$

Defining γ_t as real debt devaluation due to currency depreciation,

$$\gamma_t = \left(1 - \frac{S_{t-1}}{S_t}\right)(1+i_{t-1})\,b_{t-1},$$

and imposing interest rate parity from equation (1) yields[6]

$$b_t = (1+i)\,b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - s_t. \tag{4}$$

This reveals that debt accumulates in response to expectations of depreciation which are not realized. Expectations of depreciation raise the interest rate, and when the depreciation does not occur, debt accumulates in response to the higher interest rate.

Optimization by the representative agent, together with the assumption that governments do not allow their debt to become negative in the limit, implies a government intertemporal budget constraint given by[7]

$$\lim_{T\to\infty} E_t b_{t+T}\left(\frac{1}{1+i}\right)^T = (1+i)\,b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - E_t\sum_{h=0}^{\infty} s_{t+h}\left(\frac{1}{1+i}\right)^h = 0. \tag{5}$$

[6] First, substitute for γ_t in equation (3) yielding $b_t = (1+i_{t-1})\,b_{t-1} - \gamma_t - s_t$. Then use interest rate parity to yield $(1+i_{t-1})\,b_{t-1} = \frac{(1+i)b_{t-1}}{E_{t-1}\left(\frac{S_{t-1}}{S_t}\right)}$, which implies $(1+i_{t-1})\,b_{t-1}E_{t-1}\left(\frac{S_{t-1}}{S_t}\right) = (1+i)\,b_{t-1}$. Noting that $(1+i_{t-1})\,b_{t-1}E_{t-1}\left(\frac{S_{t-1}}{S_t}\right) = E_{t-1}(1+i_{t-1})\,b_{t-1}\left(\frac{S_{t-1}}{S_t}\right) = (1+i_{t-1})\,b_{t-1} - E_{t-1}\gamma_t$. and substituting, the equation becomes $(1+i_{t-1})\,b_{t-1} - E_{t-1}\gamma_t = (1+i)\,b_{t-1}$. Solving for $(1+i_{t-1})\,b_{t-1}$ and substituting into the first equation above yields the expression in the text.

[7] Woodford (1994) derives of the constraint as an equilibrium condition for a closed economy.

Note that surprise depreciation ($\gamma_t - E_{t-1}\gamma_t > 0$) is a source of government revenue. Anticipated depreciation is not because it creates an offsetting increase in the interest rate from interest rate parity.

2.4.2 Upper Bound

The **third key assumption** is that there is an upper bound on the present value of future primary surpluses, equivalently from equation (5), on the value of debt. We motivate this assumption with the realization that taxes are distortionary such that there exists an upper bound on the present value of taxes that the government can collect.

The upper bound rules out an explosive equilibrium, in which government debt can rise forever as long as its rate of increase is less than the interest rate. Such an explosive equilibrium, allowed in work on sustainable and passive fiscal policy, satisfies the government's intertemporal budget constraint (equation 5); however, debt grows forever, eventually exceeding any upper bound.[8] In the presence of an upper bound, debt must not grow beyond its upper bound. Therefore, when debt is subject to an upper bound, fiscal sustainability requires that the model in the primary surplus and debt be dynamically stable, such that debt attains a long-run equilibrium value below its upper bound.

2.4.3 Surplus Rule

Fiscal policy is defined by the behavior of the primary surplus, which we refer to simply as the surplus.[9] We assume that the fiscal authority is able to commit to a rule. The rule

[8] With output growth, all of these restrictions can be expressed with variables defined as a fraction of output and with the interest rate defined as the real growth-adjusted interest rate.

[9]

we choose is simple and does not require specification of a fully general equilibrium model. However, any rule with fiscal risk could be used to complete the model.

The **fourth key assumption** is that fiscal policy entails risk. We assume a baseline fiscal rule in which the surplus responds positively to lagged debt service by a magnitude sufficient to allow a long-run equilibrium in which the surplus services debt at the world interest rate.[10] The baseline fiscal policy is augmented by introducing stochastic shocks. Stochastic shocks, together with the upper bound, imply risk to current fiscal policy.

The surplus rule is given by

$$s_t - s_{t-1} = \alpha \left(ib_{t-1} - s_{t-1}\right) + \nu_t \qquad \frac{i}{1+i} < \alpha < 1, \tag{6}$$

where ν_t is a bounded, stochastic disturbance representing fiscal shocks $(-\bar{\nu} \leq \nu_t \leq \bar{\nu})$. Fiscal shocks (ν_t) contain all determinants of the surplus not explicitly included in the surplus rule, many of which would be explicit if the model were placed in a full general equilibrium context. The restrictions on α assure that one root of the dynamic system in debt and the surplus is less than unity and imply persistence in the surplus $(0 < 1 - \alpha < 1)$. Persistence smooths the effects of shocks over time and is consistent with empirical evidence.

Substituting equation (6) into (4), yields a dynamic equation in debt

$$b_t - b_{t-1} = i\left(1 - \alpha\right) b_{t-1} - \left(1 - \alpha\right) s_{t-1} - \nu_t - \gamma_t + E_{t-1}\gamma_t. \tag{7}$$

The dynamic system is given by equations (6) and (7). One root is unity, and the other

By treating the surplus as determined by equation (6), we are ignoring the effect of capital gains or losses on seigniorage revenue $\left(\frac{i_t}{1+i_t}\frac{M_t}{P_t}\right)$ under the assumption that the fiscal authority can adjust the surplus to offset these. We are also assuming that the government chooses real expenditures and taxes.

[10]This requires that the change in the surplus respond to debt service with the negative of its response to the lagged surplus, such that the surplus is no longer changing when the surplus equals debt service.

is $(1-\alpha)(1+i)$, which is less than one. The model is stable around a long-run equilibrium which has a unit root. As long as initial debt is not too high, debt is expected to reach a long-run equilibrium value less than its upper bound, implying satisfaction of the government's intertemporal budget constraint subject to the upper bound. Therefore, a fiscal rule given by (6) is passive.

It is useful to compare the policy assumption to those in generation-one and two models. The fiscal policy in generation-one models, financing a constant primary deficit with declining reserves, is unsustainable with probability one. After reserves have been exhausted, the government is unable to finance the primary deficit while maintaining the fixed exchange rate. Fiscal solvency is restored with an increase in the primary surplus generated from an increase in money growth. In contrast, for the fiscal solvency model presented here, the baseline policy mix in the absence of stochastic shocks is completely sustainable, as long as initial debt is not too high. Fiscal shocks, together with the upper bound, introduce risk of unsustainability.[11]

Fiscal shocks give fiscal policy similarities to that in generation-two models. In these models, exchange rate depreciation has a stabilizing role, and policy-makers can optimally choose to abandon the fixed rate in response to a stochastic rise in unemployment. In the fiscal solvency model, the stochastic fiscal shocks represent policy responses to stochastic economic or non-economic variables. The model does not preclude these being optimal policy responses. A government facing a recession might optimally choose to let tax revenue

[11] The unit root in debt implies that although the probability of unsustainability is less than one in finite time, it equals one in infinite time. Therefore, all discussion about the probability of fiscal unsustainability being less than one should be interpreted as within finite time.

fall and spending rise, while a government facing a banking collapse might optimally choose to recapitalize banks, even though these responses could lead to insolvent fiscal positions, imminently or in the future. Additionally, a government might allow a series of small negative fiscal shocks to increase the debt over time, with the expectation that the economic or political reason creating the negative shocks would end before debt accumulated sufficiently to raise crisis risk. By 2009, some countries had allowed debt to creep up to levels for which even small future fiscal shocks could lead to crisis, while others had lower debt allowing these countries to absorb substantial future negative shocks without a crisis. Therefore, in both the generation-two model and this new fiscal solvency model, a crisis can be caused by the policy response, possibly optimal, to stochastic changes in the state of the economy. The models differ in the effect of exchange rate depreciation on the economy; macroeconomic stimulus compared with fiscal solvency restoration.

2.5 Stability and Dynamics in Equilibrium

Now, consider the dynamic behavior of debt and the surplus under the initial policy mix, whereby the monetary authority fixes the exchange rate and fiscal policy is passive following equation (6). To begin, assume that debt is far enough below its upper bound that the endogenous rational value for $E_{t-1}\gamma_t = 0$. After first solving for equilibrium under pre-crisis policy and second under post-crisis policy, we analyze the full model with stochastic switching, allowing debt to enter a neighborhood for which $E_{t-1}\gamma_t > 0$.

2.5.1 Equilibrium with Initial Policy Mix

Definition 1 *For values of debt low enough that $E_{t-1}\gamma_t = 0$, constant values for the world interest rate and price level, together with a surplus rule from equation (6) and a monetary policy setting $i_t = i$, an equilibrium is a set of time series processes for the surplus, debt, and debt devaluation, $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, such that the government's flow and intertemporal budget constraints, given by equations (5) and (7), hold, expectations are rational, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

It is useful to represent the dynamics of the debt-surpus system using phase diagrams from equations (6) and (7). The phase diagram, with shocks at their expected values of zero, is given in Figure 1.

Note that the $\Delta b = 0$ and $\Delta s = 0$ schedules lie on top of each other with $ib_t = s_t$. The upper bound on debt service, given by $i\bar{b}$ at point L, implies an upper bound on the long-run value of the surplus, given by $\bar{s}$. Current fiscal shocks (ν_t) move the system away from the $\Delta s = \Delta b = 0$ locus, say to point K. As long as $E_{t-1}\gamma_t = 0$, equations (6) and (7) can be used to show that the expected relationship between debt and surpluses along an adjustment path like KFG is given by

$$\frac{i\left(E_t b_{t+1} - b_t\right)}{E_t s_{t+1} - s_t} = \frac{i\left(1-\alpha\right) - E_t \nu_{t+1}}{\alpha + E_t \nu_{t+1}}. \tag{8}$$

When the conditional mean of future fiscal shocks is zero ($E_t\nu_{t+1} = 0$), the slope of the adjustment path is constant, as drawn in Figure 1. Current shocks have long-run effects due to the unit root.

Expected future fiscal shocks change the slope of the adjustment path such that a positive expected shock implies lower expected long-run values for the debt and surplus for given initial values. Expected future fiscal shocks do not affect the current equilibrium positions for the debt and the surplus as long as $E_{t-1}\gamma_t = 0$.

When the economy is on an adjustment path like KFG, leading to a long-run equilibrium substantially below L, passive fiscal policy permits active monetary policy to fix the exchange rate such that $\gamma_t = 0$. However, $E_{t-1}\gamma_t = 0$ requires that there be no possibility of a one-period-ahead crisis, a topic to which we turn below. Rationally-determined expectations of depreciation increase as the economy moves onto adjustment paths toward long-run equilibria closer to L, and expectations change the adjustment path, as shown below.

The upper bound on the present value of surpluses and equivalently on debt implies that adjustment paths above HL cannot represent equilibrium paths. These paths require that the present value of future surpluses be larger than their upper bound in order to service debt. Rational agents would not embark on such paths because they know the present value of surpluses necessary to service debt along those paths is infeasible, implying that they cannot expect repayment of debt at market interest rates. The government must have plans to restore fiscal solvency in the event that shocks send it toward an infeasible path. We assume that agents know those plans and use them to form expectations.

The first plan we consider is a policy of switching, whereby the fiscal authority switches to active policy and the monetary to passive. Before considering the switching model, we present equilibrium under the post-crisis policy mix.

2.5.2 Equilibrium with Post-Crisis Policy Mix

In this section, we characterize equilibrium with the policy mix after switching, active fiscal policy and passive monetary policy. Under active fiscal policy, the surplus responds to a surplus target, defined as the value of the surplus in the long-run stationary equilibrium, instead of to lagged debt. The fiscal authority chooses the surplus target on the switching date, and we specify $\hat{s}$ as the largest target they would tolerate. We show below that a government which chooses to maintain the initial policy mix for as long as possible will usually choose the target equal to $\hat{s}$. The active fiscal rule with a target of $\hat{s}$ is given by

$$s_t - s_{t-1} = \alpha\left(\hat{s} - s_{t-1}\right) + \nu_t \qquad \hat{s} < \bar{s} - \bar{\nu}. \tag{9}$$

The surplus target must be below the upper bound, and, depending on tolerance for taxes, the target could be substantially lower.[12] The additional restriction is made to assure that there is no possibility of hitting the upper bound when the surplus equals the target.

The evolution of debt can be computed using equations (4) and (9) to yield

$$\Delta b_t = b_t - b_{t-1} = ib_{t-1} - (1-\alpha)\, s_{t-1} - \alpha\hat{s} - \left(\gamma_t - E_{t-1}\gamma_t\right) - \nu_t. \tag{10}$$

The passive monetary authority chooses expected inflation with its choice of the nominal interest rate, but it looses control over the actual price level. We assume that the inflation target is zero such that it chooses the interest rate to be the world value. When the surplus is low enough there is no possibility of debt crossing the upper bound, the zero inflation target implies that $E_{t-1}\gamma_t = 0$.

Definition 2 *For values of the surplus low enough that $E_{t-1}\gamma_t = 0$, constant values for the world interest rate and price level, together with a surplus rule from equation (9) and a monetary policy setting $i_t = i$, an equilibrium is a set of time series processes for the surplus, debt, and debt devaluation, $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, such that the government's flow and intertemporal budget constraints, given by equations (5) and (7), hold, expectations are rational, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

[12] A government could feasibly raise more taxes, but they might choose not to do so.

The model with active fiscal policy and passive monetary policy is given by equations (9) and (10). The phase diagram, with shocks at their expected value of zero, is given in Figure 2. This is a saddlepath-stable model in which there are debt-surplus pairs for which the present-value of debt explodes in the limit, violating the upper bound on debt.[13] To assure equilibrium, there must be one jumping variable to keep the system on the saddlepath, labeled SP, leading to long-run values for debt and the surplus at point F. The monetary authority's inflation target restricts $E_{t-1}\gamma_t = 0$, but places no restrictions on γ_t. Therefore, γ_t jumps, implying jumps in b_t from equation (4), to keep the system on the saddlepath. Stochastic and symmetric[14] surprise appreciations and depreciations $(\gamma_t - E_{t-1}\gamma_t)$, finance positive and negative stochastic surplus shocks. This is the mechanism in the FTPL. Debt devaluations and revaluations are symmetric in response to symmetric fiscal shocks, implying that policy shocks do not generate systematic revenue in the post-crisis equilibrium. This contrasts with post-crisis policy in the original generation-one model in which systematic money growth and inflation generates seigniorage.

Denoting the present-value of expected future surplus shocks by $V_t = E_t \sum_{h=t}^{n} \frac{\nu_h}{(1+i)^{h-t}}$, we solve the model in the appendix allowing the relationship between debt and the surplus along the saddlepath to be expressed as

$$b_t = \left(\frac{1}{\alpha + i}\right)\left[\frac{1+i}{i}\alpha\hat{s} + (1-\alpha)\, s_t + (1+i)\, V_t\right], \tag{11}$$

where $V_t = 0$ along SP in Figure 2. The larger the surplus target, the higher is the value of

[13] One root is $1+i$ and the other is $1-\alpha$.

[14] In the active fiscal policy regime with no possibility of policy switching, prices jump in response to fiscal shocks to keep the surplus and debt on the saddlepath. With rational expectations, price surprises must be symmetric. See Daniel (2007).

debt along the saddlepath.

From equation (11), an increase in expected future government spending, denoted by $V_t < 0$, reduces the equilibrium value of debt. Assume that the system is at point A along SP in Figure 2 at time t, when agents begin to expect an increase in spending at time $h > t$. Therefore, $E_t \nu_h < 0$. This expected future spending shock requires additional revenue to assure intertemporal budget balance. Price surprises generate revenue, whereas anticipated price changes do not, implying that a price increase on the date that expenditures rise cannot generate the necessary revenue. Therefore, the price must jump on the date on which the future spending becomes anticipated. The jump reduces real debt from point A to point B in Figure 2. Debt and the surplus then follow the unstable arrows of motion, with debt falling and the surplus increasing to reach point C on date h. The increase in spending and the associated increase in debt on date h return the system to SP at point D.

The upper bound on debt implies that post-crisis policy mix is not sustainable with probability one. Positive surplus shocks could send debt along the saddlepath above its upper bound. This would require a plan for reverse-switching, not explicitly considered here. We do assume in equation (9) that $\hat{s} \leq \bar{s} - \bar{\nu}$ to assure that reverse switching cannot occur within one period after reaching $\hat{s}$.[15]

3 Exchange Rate Crisis with Policy Switching

The previous section characterized equilibrium, first under passive fiscal and active monetary policy (Regime 1), and, second, under active fiscal and passive monetary policy (Regime 2).

[15]This assures that $E_{t-1}\gamma_t = 0$ along the saddlepath as s approaches $\hat{s}$ from the left.

In both cases, we assumed that initial values were low enough that the upper bound played no role. The upper bound on debt implies that the policy mix in Regime 1 does not assure fiscal solvency for all possible fiscal shocks. In this section we consider an equilibrium in which policy is initially in Regime 1 with plans to switch to Regime 2 once equilibrium in Regime 1 is no longer feasible. The policy switching model can be viewed as a dynamic "Fiscal Theory of the Price Level" model of exchange rate crises.[16]

We assume that the government maintains both its commitments to a fixed exchange rate and to the passive fiscal rule until equilibrium under the initial policy mix is no longer feasible. A similar assumption in generation-one crisis models, that money-financed government spending continues until no longer possible, has been criticized as suboptimal. Rebelo and Vegh (2002) have shown that it is optimal to abandon the fixed exchange rate regime as soon as failure becomes inevitable. We demonstrate below that under the policy assumed in this model, a crisis occurs as soon as it becomes inevitable, satisfying the optimality criteria in Rebelo and Vegh (2002). Moreover, a country, which continues to follow the rule when crisis probability becomes positive, could receive favorable shocks and avoid the crisis.

To allow Regime 1 to be initially viable, but subject to risk, we assume that the initial values satisfy $s_{t-1} < ib_{t-1} < \hat{s}$. This implies that debt is rising, and that debt service is low enough to place the system below the saddlepath to $\hat{s}$.

[16]Daniel (2001b) presents a fiscal theory exchange rate crisis model in which an unanticipated shock causes the solvency crisis. Uribe(2006) presents a fiscal theory model in which the role of devaluation is to eliminate hyperinflation, not restore fiscal solvency. Sims (1997) presents an FTPL switching model with switching and exchange rate crisis conditioned on the level of government debt, and Cochrane (2003, 2005) notes that the FTPL can explain a currency crisis.

3.1 Equilibrium with Switching

Definition 3 *Given constant values for the world interest rate and price level, an upper bound on the long-run value of debt, a policy mix, defined by a surplus rule from equation (6) and a monetary policy fixing the exchange rate, which the government will maintain as long as possible, and plans for policy-switching in the event that the initial policy mix becomes infeasible, an equilibrium is a set of time series processes for the surplus, debt, and debt devaluation,* $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$*, such that the government's flow and intertemporal budget constraints, given by equations (7) and (5), hold, expectations are rational, debt does not exceed its upper bound, and world agents expect to receive the return on assets determined by interest rate parity, equation (1).*

The pre-crisis equilibrium differs from that presented in Section 2.5.1 as debt nears its upper bound because $E_{t-1}\gamma_t$ becomes positive. Interest rate parity implies that the interest rate increases as expected depreciation rises, causing debt to increase more rapidly than along the adjustment path in equation (8). The dynamics leading up to the crisis and the timing of the crisis hinge critically on the behavior of expectations. Expectations depend on assumptions about the policy response.

We assume that **when faced with a crisis in which commitment to the initial policy mix creates a sudden stop in lending at time t, the government announces a policy switch at time $t+1$. The fiscal authority chooses the surplus target in the passive fiscal rule to allow exchange rate depreciation, but never appreciation.**

Since the post-crisis system in debt and the surplus is saddlepath stable, the distance between the value of debt along the saddlepath to the surplus target and its current value must be zero at time t, immediately after the policy switch. This is necessary to rule out explosive behavior of debt. If a crisis occurs with desired debt above the $\hat{s}-$saddlepath, the fiscal authority chooses $\hat{s}$ as the surplus target. The exchange rate depreciates, reducing the real value of debt onto the saddlepath leading to $\hat{s}$. The authorities are willing to choose the target equal to the largest they can tolerate ($\hat{s}$) because this choice allows them to maintain the fixed exchange rate for as long as possible, as shown below.

When a crisis occurs with debt below the $\hat{s}-$saddlepath, the fiscal authority avoids the reduction in government revenue, which would be associated with appreciation, by allowing the surplus target to fall to $\hat{s}' < \hat{s}$. This shifts the equilibrium saddlepath downward such that debt is on the $\hat{s}'$-saddlepath, and the exchange rate does not change.

3.2 Exchange Rate Depreciation and Expectations

To solve for exchange rate expectations, assume that fiscal shocks are determined by a bounded, symmetric, mean-zero distribution and that agents know the fiscal response to a sudden stop in lending. Since post-crisis debt must be on the saddlepath, the exchange rate must depreciate when desired debt under passive fiscal policy, given by b_t in equation (7), is above the saddlepath to $\hat{s}$. Using equations (11) and (7), the distance between the $\hat{s}-$saddlepath value of debt and its current value at time t can be expressed as

$$\Omega_t = (\gamma_t - E_{t-1}\gamma_t) + \frac{1+i}{\alpha+i}\left[\delta_{t-1} + \nu_t\right], \tag{12}$$

where δ_{t-1} is the state variable determining this distance at time t and is given by

$$\delta_{t-1} = \frac{\alpha}{i}\left(\hat{s} - ib_{t-1}\right) + (1-\alpha)\left(s_{t-1} - ib_{t-1}\right). \tag{13}$$

The state variable determining the time t distance is known at time $t-1$, and therefore receives a $t-1$ subscript. It is increasing in the values for $\hat{s}$ and α.[17] In equation (12), the

[17] This is because $s_{t-1} < ib_{t-1} < \hat{s}$, in the equilibrium with risk in Regime 1.

distance depends on the expectation of depreciation, on realizations of the surplus shock and depreciation, and on δ_{t-1}.

We define a shadow value of depreciation, analogous to the shadow value of the exchange rate in generation-one currency crisis models (Flood and Garber 1984). The shadow value of depreciation represents the reduction in the value of debt needed for the economy to reach the saddlepath to $\hat{s}$, equivalently to set $\Omega_t = 0$. The shadow value is positive when desired debt is above the saddlepath and negative when debt is below.

Definition 4 *The* ***shadow value*** *of debt devaluation at time t, $\tilde{\gamma}_t$, is defined as the value of γ_t for which $\Omega_t = 0$.*

Setting $\Omega_t = 0$ in equation (12) and solving yields

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \frac{1+i}{\alpha+i}\left(\delta_{t-1} + \nu_t\right). \tag{14}$$

Expectations of depreciation raise the interest rate, increasing debt, increasing actual depreciation needed to set the real value of debt on the saddlepath.

We determine crisis timing as a consequence of solving for expected depreciation. To solve for expected depreciation, **assume that agents believe that a lending crisis will occur if** $\tilde{\gamma}_t \geq 0$.[18] We prove that this assumption is consistent with a rational expectations equilibrium below. The government responds to the crisis with policy-switching, and when $\tilde{\gamma}_t > 0$, currency depreciation places the system on the saddlepath to $\hat{s}$. When $\tilde{\gamma}_t = 0$, the system is on the saddlepath without depreciation. This implies that the actual value for

[18]This does not rule out a crisis with $\tilde{\gamma}_t < 0$. Since such a crisis would not entail depreciation, equation (15) below is accurate.

depreciation in period t is given by

$$\gamma_t = \max\{\tilde{\gamma}_t, 0\} = \max\left\{E_{t-1}\gamma_t - \frac{1+i}{\alpha+i}(\delta_{t-1}+\nu_t), 0\right\}, \tag{15}$$

where we have used equation (14) to substitute for $\tilde{\gamma}_t$.

Proposition 1 *When the state variable determining the distance to the saddlepath at time t is greater than or equal to zero $(\delta_{t-1} \geq 0)$, solutions for expected and actual depreciation $(E_{t-1}\gamma_t, \gamma_t)$ exist under the initial policy with plans for switching. When $\delta_{t-1} < 0$, equilibrium solutions for expected and actual depreciation under initial policy do not exist, implying that equilibrium dynamics must bound the system away from negative values for δ_{t-1}.*

The proof is contained in the appendix. For agents to lend, they must be compensated for expectations of depreciation by a higher interest rate. Proposition 1 implies that there is a high enough interest rate when $\delta_{t-1} \geq 0$, but not otherwise. This result is most easily explained with a phase diagram. Figure 3 superimposes the $\Delta s = 0$ curve and the saddlepath for the active-fiscal-policy system on the phase diagram for the passive-fiscal-policy system. Assume s_{t-1} lies to the left of $\hat{s}$.

The requirement that $0 \leq \delta_{t-1}$ implies that the ex ante value for b_t, that is, the value before accounting for expectations $(E_{t-1}\gamma_t)$, current fiscal shocks (ν_t), and actual depreciation (γ_t) is below the saddlepath (equations 12 and 13). The assumption that fiscal shocks are bounded in magnitude implies that when $\delta_{t-1} > \bar{\nu}$, no shock could send the system above SP. However, for $\delta_{t-1} < \bar{\nu}$, some values for fiscal shocks could send debt above the saddlepath. If b_t lies above the saddlepath after accounting for expectations and the current fiscal shock, then $\gamma_t = \tilde{\gamma}_t > 0$, requiring exchange rate depreciation to place debt on the saddlepath in period t. Since the exchange rate depreciates for some values of the shock and appreciates for none, expected depreciation rises above zero $(E_{t-1}\gamma_t > 0)$ once $\delta_{t-1} < \bar{\nu}$. Positive expected depreciation increases the interest rate, raising debt and reducing the distance to the saddlepath. Therefore, once expectations of depreciation become positive, realizations of fiscal shocks must be more favorable than average to keep debt from rising above the saddlepath. Equivalently, once expectations become positive, the probability of a near-term crisis rises to something over fifty percent. However, with sufficiently favorable shocks, the crisis could be avoided.

Once debt has risen so much that it lies on the saddlepath $(\delta_{t-1} = 0)$, expectations of depreciation are so high that one-period-ahead depreciation could be avoided only for the most favorable fiscal shock.[19] Depreciation equals its shadow value. Using equation (15) to solve for depreciation when $\delta_t = 0$ yields

$$\gamma_t = \tilde{\gamma}_t = E_{t-1}\gamma_t - \left(\frac{1+i}{\alpha+i}\right)\nu_t \geq 0 \tag{16}$$

The sign restriction is required since depreciation must be greater than or equal to zero for any realization of ν_t, including its upper bound value of $\bar{\nu}$. This yields $E_{t-1}\gamma_t \geq \left(\frac{1+i}{\alpha+i}\right)\bar{\nu}$. Therefore, there are multiple equilibria in which expectations of depreciation and actual depreciation can be arbitrarily large. To verify, take the expectation of equation (16) to yield an identity in the expectation.

A value of $\delta_{t-1} < 0$ implies that b_t is above the upper bound path before accounting for expectations or current fiscal shocks. All fiscal shocks send the system above the boundary path such that the probability of depreciation is unity. However, taking expectations of

[19] See the appendix for the proof.

equation (15), when the probability of depreciation is unity, yields

$$E_{t-1}\gamma_t = E_{t-1}\gamma_t - \frac{1+i}{\alpha+i}\delta_{t-1.} \tag{17}$$

With $\delta_{t-1} < 0$, there is no solution for $E_{t-1}\gamma_t$ which satisfies equation (17). Rationally anticipated policy switching cannot restore fiscal solvency because actual depreciation cannot equal itself plus a negative gap. Therefore, in equilibrium, the dynamics must bound the system away from positions for which $\delta_{t-1} < 0$. This criterion determines crisis timing.

Proposition 2 *A crisis occurs in the first period for which $\delta_t \leq 0$. Policy-switching restores equilibrium and allows government borrowing.*

The proof is contained in the appendix. Consider the dynamics leading up to a crisis, when the crisis is caused by current surplus shocks, using the phase diagram in Figure 3. When the system is far below SP, say at point A with $\delta_{t-1} > \bar{\nu}$, no shock could send the system above SP, and the arrows of motion for the passive-fiscal-policy system govern. Consider the feasibility of a position like C. As the system approaches the saddlepath, δ_{t-1} falls below the upper bound on fiscal shocks ($\bar{\nu}$), and the market begins to anticipate depreciation, given by equation (24). This anticipation raises the interest rate to incorporate the increase in expected inflation from equation (1). The monetary authority allows the interest rate to rise to keep the current exchange rate fixed. Therefore, debt is expected to increase more quickly than implied by the locus CD, reaching SP at a point like E.

A crisis occurs when commitment to the original policy mix convinces agents not to lend. When $\delta_t < 0$, no rationally expected value for the exchange rate could restore fiscal solvency and allow agents to expect market rates of return, implying that agents refuse to lend into this position. When $\delta_t = 0$, there are multiple equilibrium values for expected depreciation, conditional on policy switching with probability one. Therefore, commitment to the original policy would prevent lending, but policy-switching permits it. This implies that a crisis with policy-switching occurs once $\delta_t \leq 0$.

When $\gamma_t = 0$, as necessary in equilibrium in Regime 1 with the monetary authority pegging the exchange rate, the evolution of δ_t is given by[20]

$$\delta_t = -(\alpha + i)\,\tilde{\gamma}_t - \alpha\,(\hat{s} - ib_t)\,. \tag{18}$$

Equation (18) reveals that there are two ways a crisis could occur. Since $\hat{s} - ib_t > 0$ in the relevant region, a value for $\tilde{\gamma}_t > 0$ is sufficient to imply $\delta_t < 0$. Therefore, a negative surplus shock could send the system above the $\hat{s}-$saddlepath in period t. Agents will not lend into this position, and regime switching with depreciation brings the system to the saddlepath.

Alternatively, the dynamics for the surplus and debt under passive policy could imply that debt next period in the absence of the regime switch would travel above the $\hat{s}-$saddlepath such that $\delta_t \leq 0$, but $\tilde{\gamma}_t < 0$. This is possible since the slope of the adjustment path (equation 8) is less than the slope of the saddlepath (using equation 11).[21] Agents will not lend into this position since no rationally-expected value for the future depreciation could satisfy interest rate parity and restore fiscal solvency. Rewriting the distance to the saddlepath as a function

[20]Use equations (4), (12), and (13), to yield

$$\delta_t = -(\alpha + i)\,E_{t-1}\gamma_t + (1+i)\,(\delta_{t-1} + \nu_t) - \alpha\,(\hat{s} - ib_t)\,.$$

Then substitute equation (14) for $\tilde{\gamma}_t$.

[21]However, since the slopes are not very different for reasonable values of i and α, and since expectations of depreciation cause the debt to increase more quickly than along the passive fiscal-policy adjustment path, the region in which a crisis without depreciation could occur is small. In simulations, even when α is very small yielding the largest difference in slopes, depreciation is required over 95% of the time.

of $\hat{s}' < \hat{s}$ yields

$$\Omega_t = -E_{t-1}\gamma_t + \frac{1+i}{\alpha+i}\left[\frac{\alpha}{i}\left(\hat{s}' - ib_{t-1}\right) + (1-\alpha)\left(s_{t-1} - ib_{t-1}\right) + \nu_t\right]. \tag{19}$$

The value for $\hat{s}'$ is chosen to set $\Omega_t = 0$ in equation (19), shifting the saddlepath downward until the real value of debt without depreciation is on the lower saddlepath, leading to a lower long-run value for debt. The reduction in the surplus target requires an increase in current surpluses to reduce long-run debt, thereby raising the present value of future surpluses enough to restore fiscal solvency. When a sudden stop occurs with debt below the saddlepath, there is no exchange rate change.

As long as $\delta_t > 0$, the probability of receiving near-term shocks, favorable enough to avoid a crisis, is positive, and no crisis occurs. Once $\delta_t \leq 0$, there is no possibility of receiving near-term shocks favorable enough to avoid the crisis, and the crisis occurs immediately.

Since δ_{t-1} is increasing in the value of the surplus target from equation (13), a fiscal authority which wants to maintain the initial policy mix as long as possible will choose the surplus target equal to $\hat{s}$ for all crises with depreciation. A smaller value would reduce the distance to the post-crisis saddlepath, increasing the probability of a crisis each period.

After the regime switch, capital gains and losses on debt due to exchange rate changes are symmetric, implying that expectations of inflation and exchange rate change return to their original values of zero. Therefore, in contrast to the post-crisis equilibrium in generation one models, real money demand is unchanged, implying that any effects of the currency crisis which tend to reduce the money supply must be sterilized.[22]

[22]It is possible to consider alternative post-crisis inflation targets with different implications for sterilization as in Daniel (2001).

3.3 Shocks Other Than Current Fiscal Shocks

The shadow value of debt devaluation, $\tilde{\gamma}_t$, is affected by anything which changes the post-crisis position of the saddlepath value of debt relative to its current value. An increase in $\tilde{\gamma}_t$ can cause a crisis if the increase is large enough that $\tilde{\gamma}_t$ becomes positive, or can increase the probability of one. Assume that the economy is in Regime 1 with $s_{t-1} < ib_{t-1} < \hat{s}$.

3.3.1 Expected future fiscal shocks

Consider the effect of an increase in expected future government spending.[23] From equation (11), this reduces the equilibrium value of debt under post-crisis policy, modifying the expression for the distance to

$$\Omega_t = (\gamma_t - E_{t-1}\gamma_t) + \frac{1+i}{\alpha+i}\left[\delta_{t-1} + \nu_t + V_t\right].$$

An increase in expected future government spending is represented by negative expected present-value surplus shocks $(V_t < 0)$. The distance (Ω_t) falls, and the shadow value for γ_t rises.

3.3.2 Confidence and the parameters of the surplus rule

We have assumed that the parameters governing fiscal policy are known. A reduction in $\hat{s}$ shifts $\Delta s = 0$ down, shifting the saddlepath down. A reduction in α increases the slope of the saddlepath without increasing long-run equilibrium values. Both reduce δ_{t-1} from equation (13), thereby raising $\tilde{\gamma}_t$ from equation (14).

However, going forward, these parameters are not actually known. Agents base their

[23] In the model, the current increase in expected future spending is totally unanticipated.

expectations about the solvency of fiscal policy on expectations of these parameters. An economic or political crisis could reduce confidence in the government's ability to raise taxes in response to an increase in debt, reducing α, or to generate taxes necessary to service as high a level of debt as before, reducing $\hat{s}$. Therefore, a loss of confidence could create a fiscal solvency crisis.

4 Alternative Policy Responses to a Crisis

Policy-switching is only one possible policy response to a crisis. In this section, we briefly consider others.

4.1 Devalue and Repeg without Policy-Switching

The government could respond to a lending crisis with a devaluation, repegging the exchange rate at a lower value to reach the adjustment path toward its surplus target, without any fiscal policy change. The target could be lower than the upper bound, say at $\hat{s}$ in Figure 1. When $\hat{s} < \bar{s}$, adjustment paths above those leading to $\hat{s}$ cannot be equilibrium paths because positions along them would imply a negative ex ante distance to the adjustment path. This implies that the relevant adjustment path becomes KFG, leading to the surplus target $\hat{s}$.

Let δ_{t-1} be redefined as the state variable determining the distance between the target value for debt, given by $\hat{s}/i$, and the current expectation of its long-run value under passive fiscal policy from equation (21). With the monetary authority maintaining the fixed exchange rate for as long as possible, the interest rate rises as this distance shrinks, assuring interest rate parity from equation (1). This policy response implies the third proposition.

Proposition 3 *A policy in which the government devalues to place the system on the adjustment path toward $\hat{s}$ and repegs at the lower rate without fiscal reform will fail next period with probability one.*

The proof is in the appendix. Since the policy response sets $\delta_t = 0$, there are multiple equilibria with arbitrarily high expectations of devaluation and accompanying high interest rates. Additional devaluation is needed each period to set $\delta_{t+i} = 0$, implying that markets remain turbulent. Given sustained post-crisis turbulence, it would be difficult to make a case that this policy represents an optimal response.

4.2 Default

The government could plan to respond to a crisis by reneging on its no-default commitment. Both default and devaluation reduce the real value of outstanding debt, moving the system toward the $\hat{s}-$saddlepath. The larger the default, the smaller the equilibrium devaluation. However, given that default typically takes time to resolve and given that agents usually do not know the magnitude of default, expectations about the magnitude could be volatile, implying a volatile exchange rate until a value for default is finalized. Since default solves the same fiscal solvency problem as devaluation, default and devaluation can occur together.[24]

4.3 IMF Loan

Assume that the country plans to resolve the crisis by securing an IMF loan to replace the private market source of loans it looses in a crisis. To simplify the presentation and contrast this policy with those preceding it, we assume there is no accompanying debt devaluation

[24] For an analysis of default as a response to the crisis, see Daniel and Shiamptanis (2009). The interest rate parity equation must be modified such that the world interest rate is equated with the expected return on the domestic asset, conditional on the possibility of default.

either through depreciation or default. An IMF loan with debt devaluation would be analyzed as a combination of the two policy responses.

The IMF is willing to make the loan when the private market is not because the IMF can mandate fiscal policy change as a condition for receiving the loan. IMF programs for countries with fiscal problems usually require an increase in the value of the government surplus for a specific period of time. We model this as an increase in the mean of ν_t for a specific period of time. In Figure 1, this flattens the adjustment path, leading to a lower expected value for the long-run surplus, restoring fiscal solvency. The government's intertemporal budget constraint is restored because the present value of future surpluses rises, not because the real value of debt falls.

However, success requires confidence in the stronger fiscal policy. Past failures to comply with IMF mandates could weaken confidence in the government's ability to deliver the present-value surpluses necessary to service debt. Additionally, governments might not be willing to restore fiscal solvency solely through increased present-value surpluses when other methods of raising revenue, including depreciation and default, are available.

5 Model Applied to Recent Currency Crises

In this section we illustrate the implications of the fiscal solvency model for several recent currency crises not explained well by the standard generation-one model. Fiscal solvency remains essential in the explanation but future seigniorage is not.

5.1 Argentina 2001: Current Fiscal Shocks and Loss of Confidence

The first crisis we consider is the Argentine crisis in 2001. This crisis cannot be explained by a generation-one model since the currency board prevented money-financed government deficits and made a total run on reserves impossible. Yet, increasing government deficits and debt did precede the crisis, and the crisis was characterized by both currency depreciation and sovereign default, highlighting the role of fiscal problems. Currency depreciation reduced the real value of government debt, even though most of it was denominated in dollars, due to the policy of pesoization, whereby dollar-denominated debt was converted to peso-denominated debt after the eruption of the crisis. According to the IMF's assessment, "eventually the required primary surplus became implausibly large, particularly in relation to the political system's ability to deliver....By 2001, almost no strategy would have succeeded without a sovereign debt restructuring that reduced the present value of Argentina's public debt burden." (IMF 2003, p. 67) We argue that the sudden stop in lending, due to the "implausibly large" required primary surplus, forced policy change which allowed the debt devaluation necessary to return the Argentine government to fiscal solvency.

How did Argentina's debt grow so large, relative to its ability to service it? Figure 4A shows the behavior of the primary surplus relative to GDP and debt relative to GDP over the decade of the 1990's. After the introduction of the currency board in 1991, strong fiscal surpluses brought debt down. By the middle of 1993, interest rates, shown in Figure 4B, were stable in the 10 percent range. However, both the Tequilla recession in 1995 and the recession which began with the Russian default in 1998 contributed to the upward path for debt from the end of 1993. Interest rates were relatively stable, with the exception of the Tequilla crisis in 1995, until they began to anticipate the crisis at the end of 2001. A sudden stop in lending at the end of November 2001 forced a policy response to restore fiscal solvency.[25] The response included elimination of the currency board to allow currency depreciation, pesoization of government debt, and ultimately, outright default on 75 percent of remaining government dollar-denominated liabilities.[26]

We simulate the policy-switching model to understand how the crisis erupted. For the simulations, we assume that the values for the primary surplus and debt in the model represent ratios to GDP, implying that the interest rate variable is the growth-adjusted real interest rate. We assume that the growth-adjusted real interest rate is .02, based on real interest rates of about 5-6% and reasonable expected growth rates of 3-3.5% (IMF 2003, p.12). We let the long-run value for the surplus target be 1.5% of GDP, larger than any surplus achieved in the 1990's, although not larger than the IMF surplus targets for the short run. At this interest rate, $\hat{b}$ =75% of GDP.[27] We assume that fiscal shocks have a uniform distribution and let the upper bound on fiscal shocks be 2% of GDP to accommodate the large fiscal shock in 2001.[28]

We need a value for α in equation (6), under the fiscal policy implemented with the introduction of the currency board in 1991. Figure 4C plots the change in the primary

[25]The event was a run on private sector deposits of more than US$ 3.6 billion (6 percent of the deposit base) during November 28-30 (IMF 2003, p. 61).
[26]This occurred in 2005 (IMF 2005, p. 13).
[27]Krueger (2002) argues that the upper bound on debt for a country like Argentina, which has difficulty raising taxes, is much lower than the upper bound for an industrialized country.
[28]We use the residuals from the estimate of equation (6) to calibrate the upper bound on fiscal shocks, $\bar{\nu}$. When we estimate the fiscal rule, using only the last three years of data, the residual in 2001 is -0.0188.

surplus and the excess of debt service over the primary surplus for the decade of the 1990's, using annual IMF data[29] on the primary surplus and debt for the consolidated public sector, including federal and provincial governments, as a fraction of GDP. The series do move together, implying a positive α as required for fiscal policy to be passive. Fiscal policy seemed to change toward the end of the decade, becoming less responsive. To characterize initial policy, we estimate equation (6) for the period 1991-1998, and obtain a value for α of 0.61.[30] The estimate of α is consistent with the hypothesis that fiscal policy was initially sustainable and passive.

To determine the probability of an exchange rate crisis due to random fiscal shocks under this initial policy, we simulate the model using the above parameter values and 1991 values for debt/GDP, and the primary surplus/GDP. The simulation algorithm and the results from this simulation and others are presented in Tables 1 and 2, respectively.

With debt and the surplus at their 1991 values and parameters for fiscal policy given as above, the results of simulations, presented on the first line of Table 2, reveal that Argentina's initial fiscal policy was safe, with no probability of a currency crisis in twenty years. Perhaps Argentina had very bad luck and fiscal shocks were much more negative than random draws from a uniform distribution would imply. By the end of 2000, debt had increased to 50.9% of GDP. However, simulations with year 2000 values for the debt and surplus, presented on

[29] We used IMF (2003) for values from 1992 - 2001 and Krueger (2002) for 1991. They overlap from 1992 and are similar. We used the more recent data under the hypothesis that it contained more recent revisions and was therefore more accurate.

[30] The equation is estimated without a constant, as required by equation (6), when the mean of surplus shocks is zero. The estimate for α is statistically significant at the 1% level, and the R^2 is .39, not too bad for an equation in first differences with 7 observations! The addition of a constant raises the standard error of the equation, and the constant is not significant at any conventional level.

line 6, show a tiny increase in the probability of a crisis over twenty years (0.6%). Therefore, the actual string of negative fiscal shocks, which increased debt, is not sufficient to explain the crisis. To explain the crisis, we must consider the effect of the recession, which began in 1998, on confidence in the fiscal authority's ability to deliver the taxes necessary to carry out its original fiscal rule.

It is reasonable to argue that the 1998 recession, together with persistent failures to meet surplus targets (IMF 2003), weakened both the fiscal stance as well as confidence in fiscal policy. Particularly in 1999 and 2001, the surplus failed to respond as much to debt service relative to the surplus, as in earlier periods.[31] Fiscal policy from 1999-2001 is more consistent with a value of $\alpha = .13$.[32] Additionally, a crisis of confidence is likely to have lowered the public's expectation of the largest surplus which the government could sustain, so we assume that $\hat{s}$ falls from 1.5% to 1.25% implying a fall in $\hat{b}$ from 75% to 62.5% of GDP.[33] With reduced confidence in the fiscal stance and the debt and surplus at their 2000 values, the probability of a crisis in three years rises to 62.5% with the mean time to a crisis being a little less than one year, as illustrated on the last line of Table 2. Therefore, the model, characterized by a reduction in the surplus-responsiveness of debt and a lower upper bound on the surplus, predicts the crisis which occurred. The crisis did not occur immediately with the 1998 recession because there was still some possibility that it could

[31] Given the short sample we cannot confidently distinguish negative fiscal shocks from reduced responsiveness. Therefore, we are considering the effect on risk if agents changed their beliefs about the responsiveness, consistent with evidence.

[32] This is the regression estimate for α in equation (6) using three years of data from 1998-2001. The argument for a fall in α is further supported by an estimate of .48 over the full sample (1991-2001), compared with .61, for the first eight years (1991-1998).

[33] The IMF's post-crisis assessment (2003) shows reduced confidence in the government's ability to generate taxes.

be avoided. Simulations, using the parameters associated with reduced confidence together with the 1998 values of debt and the surplus, imply that the probability of a crisis in three years was only 22.4% with the mean time to crisis two years (line 5).

The simulations imply that reduced confidence was key to explaining the crisis. We can illustrate the importance of expectations of the weaker fiscal stance by simulating the model with 1991 values for the debt and surplus, but with $\alpha = .13$ and $\hat{b} = 62.5\%$. Had the initial 1991 stabilization program been accompanied by the weaker fiscal stance, the program would have been much riskier with crisis probability over the next twenty years rising to 69.5% with mean time to a crisis being about five years. This illustrates the importance of the strong fiscal stance, represented by a large response of the surplus to debt, for the viability of the initial stabilization program.

Therefore, the model implies that the initial monetary and fiscal reform in 1991 in Argentina carried virtually no fiscal solvency risk over a twenty year horizon. If Argentina had been able to sustain confidence in a fiscal rule, like that it initiated in the beginning of the reform, then it would have likely avoided the crisis. The recession brought not only negative surplus shocks, but also reduced confidence in fiscal policy, substantially raising the probability of a crisis. Under this weakened fiscal stance, the required "implausibly large surplus" was due both to increased debt and to reduced plausibility of raising taxes to generate surpluses previously thought feasible.

5.2 Mexico 1994-95: Devalue and Repeg without Policy Switching

The Mexican crisis was not preceded by money-financed government deficits (Calvo and Mendoza 1996). Instead, primary surpluses had exceeded 1% of GDP since 1983, and debt as a fraction of GDP had been falling since 1987. At the end of 1993, debt was 25% of GDP.[34] Mexico was at a point like G in Figure 1, moving toward a surplus target well below its upper bound. The country was not experiencing fiscal solvency problems in any standard sense and cannot be explained by a generation-one model.

The crisis began with the 15% devaluation of the peso relative to the US dollar in December 1994 (Sachs et al. 2006). Figure 5 presents end-of-month interest rates and exchange rate changes, calculated at monthly rates. As indicated by the flat behavior of interest rates leading up to the crisis and documented by newspaper accounts (Sachs et al. 2006), the devaluation was a surprise. The fiscal solvency model explains how the surprise devaluation itself could have caused the crisis.

Suppose that agents believed that the government was following passive fiscal policy,[35] and that the devaluation was designed to reduce debt onto the government's desired adjustment path toward a long-run equilibrium with lower long-run debt, setting $\delta_t = 0$. This is reasonable since the election earlier in 1994 has elicited fiscal expansion, increasing debt. The devaluation could have been used to return the debt closer to its pre-election adjustment

[34]Primary surplus data are from Banco de Mexico website: http://www/banxico.org.mx/ under "Public Finances" and other data is from IFS Statistics.

[35]Using annual data from 1984 until 1994 the estimate of α in equation (6) is .85 with a P value of .0004. A constant is included and its estimated value is 2.01 with a P value of .0004. The positive constant is consistent with the expectation of positive mean surplus changes over the horizon of the estimation. The R^2 is .85.

path.

When $\delta_t = 0$, there are multiple equilibria, in which any expectations of future devaluation can be equilibrium expectations, as long as they are high enough. And these expectations are self-fulling since the new peg fails with probability one from Proposition 3. Therefore, the model predicts that the initial devaluation would have been followed by a period of arbitrarily high interest rates and additional devaluations. Figure 5 shows that interest rates shot up after the devaluation, reflecting further expected devaluations. Additional large devaluations followed, with the cumulative December devaluation alone being greater than fifty percent.[36]

A policy to change the perception that the government would use future debt devaluation to achieve fiscal goals was necessary to end the crisis. It is reasonable to argue that both the fiscal policy changes announced in March 1995, and the large US-IMF loan announced in February restored private confidence that the authorities would not use debt devaluation to remain on the adjustment path chosen with the December devaluation. The exchange rate appreciation by the end of April confirmed that the period of arbitrarily high expectations of exchange rate depreciation had ended. Formally, the increased confidence would be modeled as an increase in $\hat{s}$, implying confidence that the government could raise the necessary taxes to service a larger value of long-run debt. The increase in $\hat{s}$ increases δ_t, reducing and possibly eliminating expectations of depreciation.

[36]The actual magnitude of the initial debt devaluation was small, since peso-denominated debt had fallen to 5.1% of GDP by the end of November, 1994 (Burnside et al. 2006), but its significance was in setting $\delta_t = 0$.

5.3 East Asia 1997: Expected Future Government Expenditures

Now, consider crises in Thailand and Korea in 1997. Neither country had fiscal solvency problems at the end of 1996. Graphs 6A and B illustrate the behavior of primary surpluses relative to GDP and debt relative to GDP from 1977 until the crisis.[37] Thailand had brought government debt down from a peak of 35% in 1986 to 4% of GDP by the end of 1996. Korean debt was low over the period and was only 2% of GDP at the end of 1996. Both countries had small positive primary surpluses. Since there were no fiscal deficits to finance with money, the generation-one model cannot explain these crises.

Burnside et al. (2001) have argued that the currency crises in East Asia were due to expected future increases in government expenditures to finance the banking crises. However, their model remains couched in terms of the generation-one currency crisis model in which an increase in future money growth and seigniorage is a necessary component in restoring fiscal solvency.[38] The fiscal solvency model can generate a currency crisis due to an increase in expected future government expenditures, irrespective of any assumptions about future monetary growth. Debt devaluation can provide the increase in revenue necessary to match the increase in expected future expenditures.[39]

The expected increases in future government spending associated with the financial crisis

[37]Data is from IFS. There are no data on either primary surpluses or interest payments on government debt for this period. Therefore, we adjusted the actual surpluses for interest on debt, using debt series and available interest rate series. For Korea, we used the money market rate, and for Thailand, we used the government bond yield.

[38]In their model the timing of the currency crisis is based on an assumption that money will take a discrete upward jump on a particular future date and then grow faster after that date.

[39]Burnside, Eichenbaum, and Rebello (2006) claim that a large fraction of the increased expenditure in Korea was financed by debt devaluation. Additionally, Jeanne and Guscina (2006) report that a very large fraction of domestic debt in an aggregate of Asian economies is denominated in domestic currency.

were large shocks, with the Burnside et al. (2001, p.1165) estimates for both countries being 30% of GDP. We calibrate the policy-switching model for both countries to determine whether an expected future fiscal shock of this magnitude could have created a fiscal solvency crisis. A crisis with depreciation would have occurred if the increase in expected future government spending raised the shadow value for debt devaluation above zero $(\tilde{\gamma}_t > 0)$. For this to have occurred in such a strong fiscal environment, in which expectations of depreciation were likely to have been zero[40], it is necessary that expected future fiscal shocks be large enough that

$$\delta_{1996} + \nu_{1997} + V_{1997} < 0,$$

where $V_{1997} = .30$ is the expected present value of government spending due to the financial crisis. Consider, first, calibration of δ_{1996}, using equation (13).

We need values for i and α. The interest rate is the growth-adjusted real interest rate, and we use a value of .02.[41] The parameter α is determined by how strongly the government adjusts the primary surplus to the gap between debt service and the primary surplus. Figures 6C and 6D reveal a positive relationship, as required for initial fiscal policy to be active. We estimate equation (6) for both countries using available data through 1997 and obtain values of α, given by 0.591 for Thailand and 0.333 for Korea.[42] This equation also provides estimates

[40]For all reasonable values for $\hat{s}$, $\delta_{1996} < \bar{\nu}$, where $\bar{\nu}$ is the largest residual in the estimate for the fiscal policy equation (6).

[41]We experimented with values of .01 and .015. The value of .02 provided marginally better fits of equation (6) for both countries. The growth-adjusted interest rate must be positive or the government does not face an intertemporal budget constraint. A lower growth-adjusted interest rate has little affect on the estimated magnitude of α. It does not have much effect on the critical value of $\hat{b}$, defined below, but since $\hat{s} = i\hat{b}$, it substantially affects the critical value of $\hat{s}$.

[42]Beginning dates are limited by the availability of data on interest rates, used to adjust the actual surplus to a measure of the primary surplus. The equation for Thailand is estimated with annual data from 1977-1997 and includes a dummy for 1988-1996. The P value for the estimate of α is .0006, and the R^2 for the equation

of ν_{1997}, -0.0130 for Thailand and 0.0004 for Korea.

The final parameter we need to calibrate δ_{1996} is the target value of the surplus, $\hat{s}$. The surplus target has the interpretation of the largest value of the surplus which the government would be willing to maintain on a permanent basis. A government with a strong tax base could maintain a larger $\hat{s}$. Another way to understand this parameter is to compute the associated long-run value of the debt as $\hat{b} = \frac{\hat{s}}{i}$.

Instead of specifying a particular value for the surplus target, we turn the problem around and solve for a critical value of the surplus target, below which there would have been a fiscal solvency crisis and above which there would not. The critical value for $\hat{s}$ sets $\delta_{1996} + \nu_{1997} + V_{1997}$ equal to zero. Using 1996 values for the debt and primary surplus for Thailand and Korea, we calculate $\hat{s}^*_{Thailand} = .0106$ and $\hat{s}^*_{Korea} = .0097$. This translates into $\hat{b}^*_{Thailand} = 0.530$ and $\hat{b}^*_{Korea} = 0.915$. That is, if agents believed that the Thai (Korean) government would be able to generate sufficient taxes to service a long-run level of debt as great as 53% of GDP (91.5% of GDP), then the baseline model says that there should have been no crisis. Alternatively, if agents believed that governments were not able to sustain taxes of this magnitude, then the model predicts the crises which occurred. These values for long-run debt relative to GDP are considerably higher than peak values in the 1980's for both countries, suggesting that agents might have thought critical values were lower. These governments were in strong financial positions, but they were struck by large negative shocks. Reasonable parameterization implies that the large shocks could have precipitated the crises.

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is .47. The equation for Korea is estimated with annual data from 1978. The P value for the estimate of α is .02, and the R^2 is .20. Neither equation includes a constant, and when a constant is added, it is not significant.

6 Conclusions

This paper provides a dynamic model of currency crises which retains fiscal solvency as the central cause, as in generation-one models. However, it replaces the initial inconsistent policy mix of the generation one model with a policy mix which fails with positive probability. And it replaces the role of seigniorage in restoring fiscal solvency with debt devaluation, created by currency depreciation. The model is the product of insights from the original generation one model, which highlights fiscal solvency as a key fundamental in exchange rate crises, and the FTPL, which allows capital gains and losses on debt to maintain fiscal solvency.

When stochastic shocks move the government's desired debt above the path leading to the long-run upper bound, agents refuse to lend, precipitating a crisis. The crisis could be caused by current fiscal shocks, which reduce the current surplus and raise desired debt, by expectations of new future fiscal commitments which reduce the expected present-value of future surpluses, and/or by a change in the current fiscal rule or the expectation of such a change. Agents will not lend and equilibrium cannot be restored until policy responds to restore expectations of fiscal solvency. One possibility is a regime switch, in which the fiscal rule becomes active and monetary policy becomes passive, as in the FTPL. This allows future price level (and exchange rate) surprises to offset stochastic surplus shocks and usually requires exchange rate depreciation in the crisis period to reduce the outstanding value of debt. A policy of devaluation to restore fiscal solvency without a change in the fiscal rule will restore equilibrium, but at the cost of arbitrarily high expected future devaluation and interest rates. This result highlights the importance of fiscal reform in restoring orderly markets after a crisis. Since currency depreciation and sovereign default both restore fiscal solvency, they can occur together.

We apply the model to explain crises in Argentina (2001), Mexico (1994), and East Asia (1997), which do not fit the stylized facts of generation one crisis models. The model can be used to attribute the Argentine crisis to negative fiscal shocks and a loss of confidence in the government's ability to raise taxes to service debt. Both currency depreciation and default on sovereign debt were used to restore fiscal solvency. The Mexican crisis can be explained by the surprise devaluation which signaled the government's willingness to use debt devaluation to keep debt along its desired adjustment path. The crises in East Asia can be attributed to large expected future government expenditures to recapitalize banks, as argued by Burnside et al. (2001).

The model advances on Daniel (2001b) by providing a dynamic "Fiscal Theory of the Price Level" model of currency crises, as well as by considering responses to currency crises other than policy switching. The FTPL model of currency crises advances on policy switching models by Sims (1997) and Davig and Leeper (2006) and Davig, Leeper, and Chung (2007) by allowing policy switching to be the endogenous response to a crisis, which resolves the crisis by restoring fiscal solvency. It highlights the danger of using devaluation without fiscal reform to restore solvency. More generally, the model provides additional support for viewing fiscal policy as a key determinant of exchange rate crises.

7 Appendix

7.1 Regime 1: Passive Fiscal Policy and Active Monetary Policy

Solutions to equations (6) and (7), treating ν_t, $\gamma_t - E_{t-1}\gamma_t$ as shocks, are given by

$$\begin{aligned} s_t &= s_{-1} + \frac{\alpha\left[ib_{-1} - s_{-1}\right]\left[1 - \left[(1-\alpha)(1+i)\right]^{t+1}\right]}{1 - (1-\alpha)(1+i)} \qquad (20) \\ &- \sum_{k=0}^{t}\left[\frac{\alpha i\left[1 - \left[(1-\alpha)(1+i)\right]^{t-k}\right](\gamma_k - E_{k-1}\gamma) + \left[i - \alpha(1+i)\left[(1-\alpha)(1+i)\right]^{t-k}\right]\nu_k^j}{1-(1-\alpha)(1+i)}\right], \end{aligned}$$

$$\begin{aligned} b_t &= b_{-1} + \frac{(1-\alpha)\left[ib_{-1} - s_{-1}\right]\left[1 - \left[(1-\alpha)(1+i)\right]^{t+1}\right]}{1 - (1-\alpha)(1+i)} \qquad (21) \\ &- \sum_{k=0}^{t}\left[\frac{\left[\alpha - i(1-\alpha)\left[(1-\alpha)(1+i)\right]^{t-k}\right](\gamma_k - E_{k-1}\gamma_k) + \left[1 - \left[(1-\alpha)(1+i)\right]^{t+1-k}\right]\nu_k^j}{1-(1-\alpha)(1+i)}\right], \end{aligned}$$

7.2 Regime 2: Active Fiscal and Passive Monetary Policy

Solutions to equations (9) and (4) under the assumption that the mean of ν_t is zero for all $t > n$, are given by

$$s_t = \hat{s} + \left[(1-\alpha)(s_{-1} - \hat{s}) + \sum_{k=0}^{t}\left(\frac{1}{1-\alpha}\right)^k \nu_k\right](1-\alpha)^t, \qquad (22)$$

$$b_t = \left(\frac{\hat{s}}{i}\right) + \left(\frac{1}{\alpha+i}\right)\left\{\left[(1-\alpha)(s_{-1} - \hat{s}) + \sum_{k=0}^{t}\left(\frac{1}{1-\alpha}\right)^k \nu_k\right](1-\alpha)^{t+1} + (1+i)^{t+1} E_t \sum_{h=t}^{n}\frac{\nu_h}{(1+i)^h}\right\}. \qquad (23)$$

The requirement that the coefficient on the explosive root be zero implies:

$$b_{-1} - \left(\frac{1}{\alpha+i}\right)\left[\left(\frac{1+i}{i}\right)\alpha\hat{s} + (1-\alpha)s_{-1}\right] - \sum_{j=0}^{t}\left(\frac{1}{1+i}\right)^{j-1}\left[\gamma_j - E_{j-1}\gamma_j + \frac{1+i}{\alpha+i}\nu_j\right] = 0.$$

7.3 Proofs

7.3.1 Proof of Proposition 1

Define a critical value for ν_t, given by ν_t^*, as the minimum value for ν_t which sets $\gamma_t = 0$. Therefore, for $\nu_t < \nu_t^*$, $\gamma_t > 0$, and for $\nu_t \geq \nu_t^*$, $\gamma_t = 0$.

Letting $f(\nu_t)$ be a bounded, symmetric, mean-zero distribution for ν_t, with bounds given by $\pm\bar{\nu}$, and using equation (15), the expectation for γ_t can be expressed as

$$E_{t-1}\gamma_t = \int_{-\bar{\nu}}^{\nu_t^*} \gamma_t f(\nu_t)\, d\nu_t = \int_{-\bar{\nu}}^{\nu_t^*} \left[-\frac{1+i}{\alpha+i} [\delta_{t-1} + \nu_t] + E_{t-1}\gamma_t \right] f(\nu_t)\, d\nu_t.$$

Defining $F(\nu_t^*)$ as the cumulative at ν_t^* and collecting terms on the expectation yields

$$[1 - F(\nu_t^*)]\, E_{t-1}\gamma_t = -\left(\frac{1+i}{\alpha+i}\right) \left[\delta_{t-1} F(\nu_t^*) + \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t \right]. \tag{24}$$

Substituting into equation (15), yields an expression for γ_t as

$$[1 - F(\nu_t^*)]\, \gamma_t = \max\left\{ -\left(\frac{1+i}{\alpha+i}\right) \left[\delta_{t-1} + \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + [1 - F(\nu_t^*)]\, \nu_t \right], 0 \right\}. \tag{25}$$

Assume that the economy is in period $t-1$. To solve for ν_t^*, define $\chi_t = \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + [1 - F(\nu_t^*)]\, \nu_t^*$. A solution for ν_t^* exists iff there exists a value for ν_t^*, satisfying $-\bar{\nu} \leq \nu_t^* \leq \bar{\nu}$, such that $\delta_{t-1} + \chi_t = 0$. When there is no solution for ν_t^*, all values of ν_t would cause depreciation.

First, we prove that $\chi_t \leq 0$ for all feasible values for ν_t^*. Let ν_t^* take on its smallest possible value of $-\bar{\nu}$, implying that $\chi_t = -\bar{\nu} < 0$. The derivative of χ_t with respect to ν_t^* is given by $1 - F(\nu_t^*)$. For $\nu_t^* < \bar{\nu}$, this is positive. Therefore, as ν_t^* rises, χ_t rises monotonically. Once ν_t^* takes on its largest possible value, given by $\bar{\nu}$, $1 - F(\bar{\nu}) = 0$, and

χ_t takes on its maximum value of zero. Therefore, $\chi_t \leq 0$ for all feasible values of ν_t^*. Since $\chi_t \leq 0$, a necessary condition for $\delta_{t-1} + \chi_t = 0$, implying existence of a solution for ν_t^*, is that $\delta_{t-1} \geq 0$.

For $\delta_{t-1} > 0$, equation (25) can be used to solve for $\nu_t^* < \bar{\nu}$, and since $1 - F(\nu_t^*) > 0$, equation (24) can be used to solve for $E_{t-1}\gamma_t$. However, for $\delta_{t-1} = 0, \nu_t^* = \bar{\nu}$ and $1 - F(\nu_t^*) = 0$. The probability of depreciation is unity. Expectations of depreciation are given by equation (16) in the text. Given a value for $E_{t-1}\gamma_t$ the actual value for γ_t is given by equation (15).

7.3.2 Proof of Proposition 2

From Proposition 1, when $\delta_t < 0$, there is equilibrium. Therefore, equilibrium dynamics must be restricted to assure $\delta_t \geq 0$. When $\delta_t < 0$, agents cannot expect market rates of interest and therefore refuse to lend into this position, precipitating a crisis in period t.

From Proposition 1 when $\delta_t = 0$, there are multiple equilibrium values for actual and expected depreciation, conditional on regime-switching with probability one. Agents would refuse to lend if they anticipated no switching. Therefore, the refusal to lend in the absence of switching precipitates the crisis.

Policy switching restores equilibrium with fiscal solvency by setting $\Omega_t = 0$, placing the system on the saddlepath associated with the post-crisis policy mix. If $\tilde{\gamma}_t \geq 0$, then $\gamma_t = \tilde{\gamma}_t$, sets $\Omega_t = 0$. If $\tilde{\gamma}_t < 0$, but $\delta_t \leq 0$, then $\hat{s}' < \hat{s}$, sets $\Omega_t = 0$ in equation (19).

7.3.3 Proof of Proposition 3

The state variable determining the distance to the adjustment path leading to $\hat{s}$ can be expressed as $\delta_{t-1} = (1-\alpha)\, s_{t-1} - \alpha b_{t-1} + \frac{1-(1-\alpha)(1+i)}{i}\hat{s}$, and the distance is given by $\Omega_t =$

$\frac{\delta_{t-1}+\nu_t+\alpha(E_{t-1}\gamma_t-\gamma_t)}{1-(1-\alpha)(1+i)}$ (Daniel and Shiamptanis 2009). After devaluation to place the system on the $\hat{s}-$adjustment path, $\delta_t = 0$, and the probability of devaluation next period is unity by Proposition 2. Expectations of devaluation $\left(E_t\gamma_{t+1}\right)$ are at least as high as $\frac{\bar{\nu}}{\alpha}$, implying high interest rates. This is because he shadow rate of devaluation is given by $\tilde{\gamma}_{t+1} = E_t\gamma_{t+1} + \frac{1}{\alpha}\left[\delta_t + \nu_t\right]$. With $\delta_t = 0$, $E_t\gamma_{t+1} \leq -\frac{\bar{\nu}}{\alpha}$. With probability one, the fixed rate fails in the subsequent period with a devaluation and in every period thereafter.

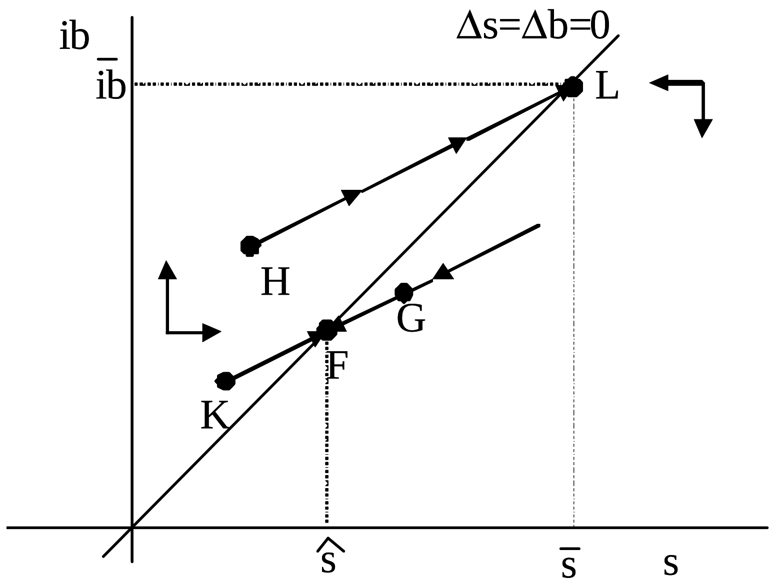


Figure 1: Passive Fiscal Policy

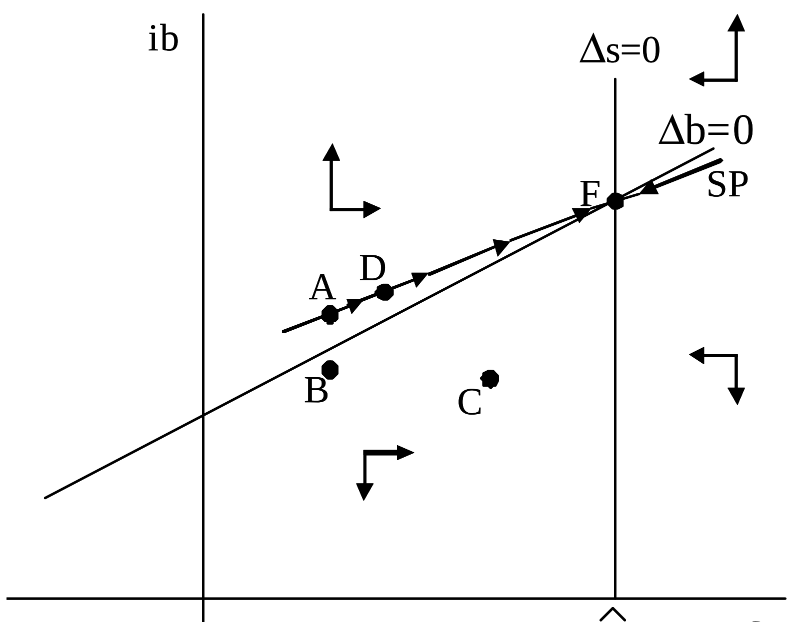


Figure 2: Active Fiscal Policy

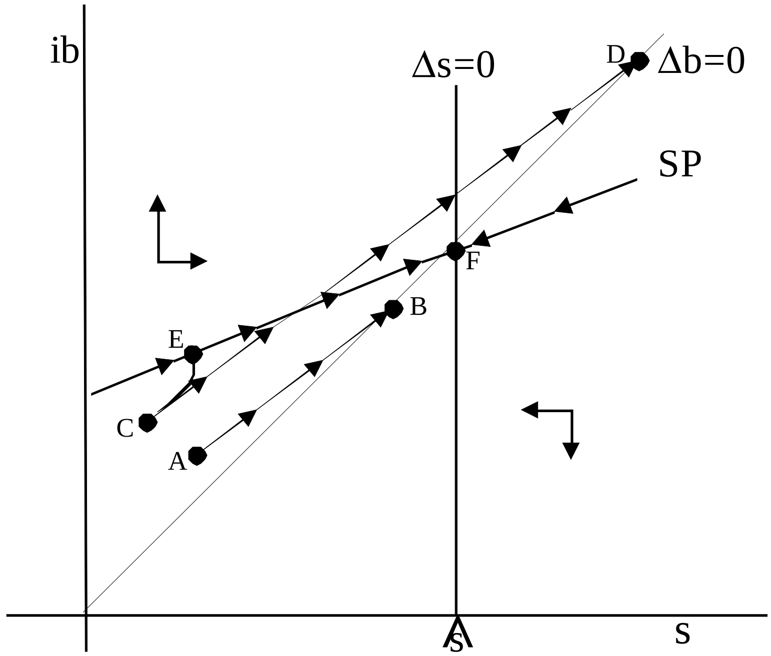


Figure 3: Switching Regime

Argentina

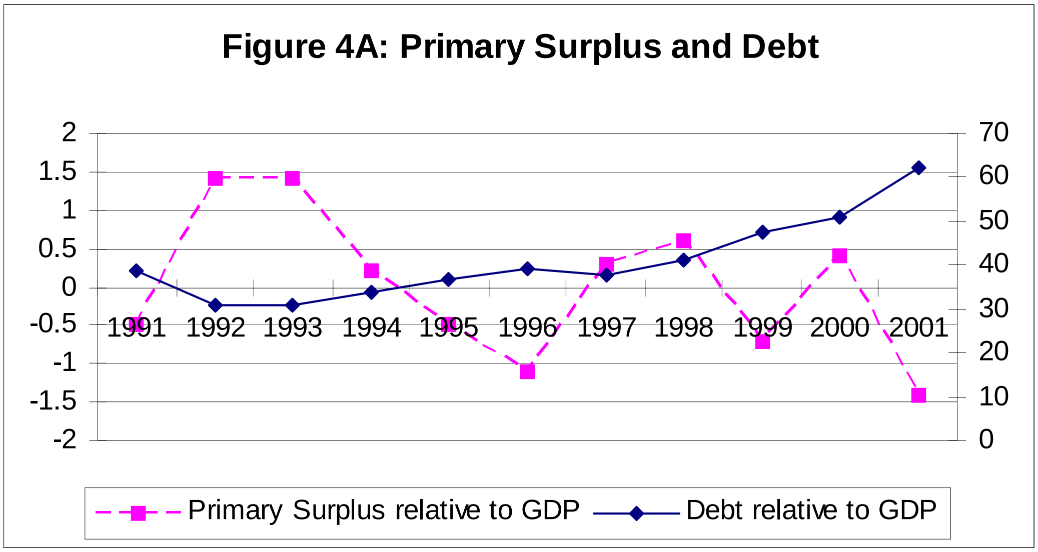


Figure 4A: Primary Surplus and Debt
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
70
60
50
40
30
20
10
0
1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001
Primary Surplus relative to GDP
Debt relative to GDP

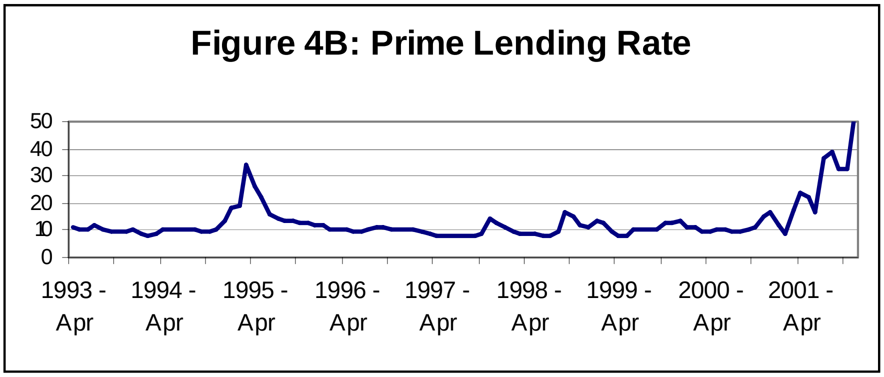


Figure 4B: Prime Lending Rate
50
40
30
20
10
0
1993 - Apr
1994 - Apr
1995 - Apr
1996 - Apr
1997 - Apr
1998 - Apr
1999 - Apr
2000 - Apr
2001 - Apr

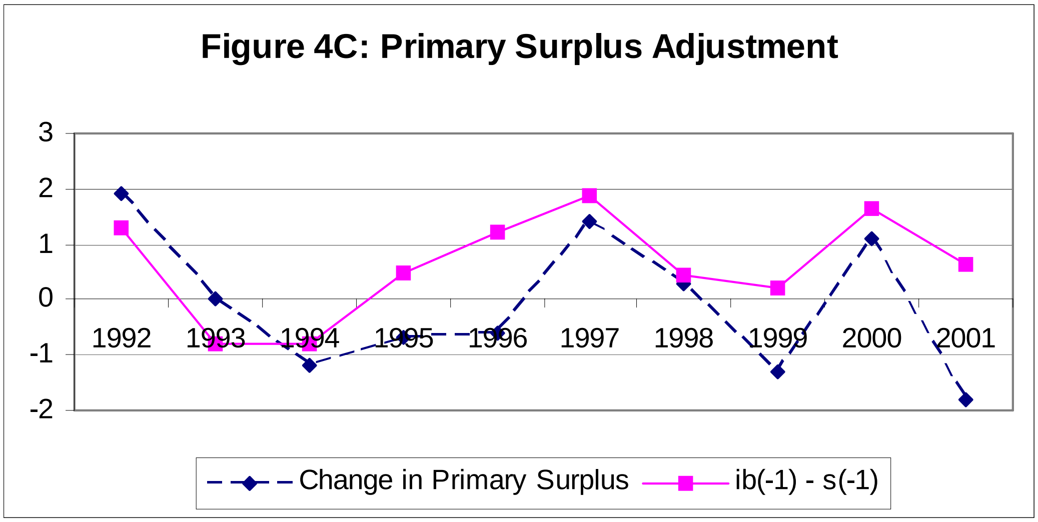


Figure 4C: Primary Surplus Adjustment
3
2
1
0
-1
-2
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001
Change in Primary Surplus
ib(-1) - s(-1)

Table 1

Simulation Algorithm: Probability of a Crisis in Twenty Years

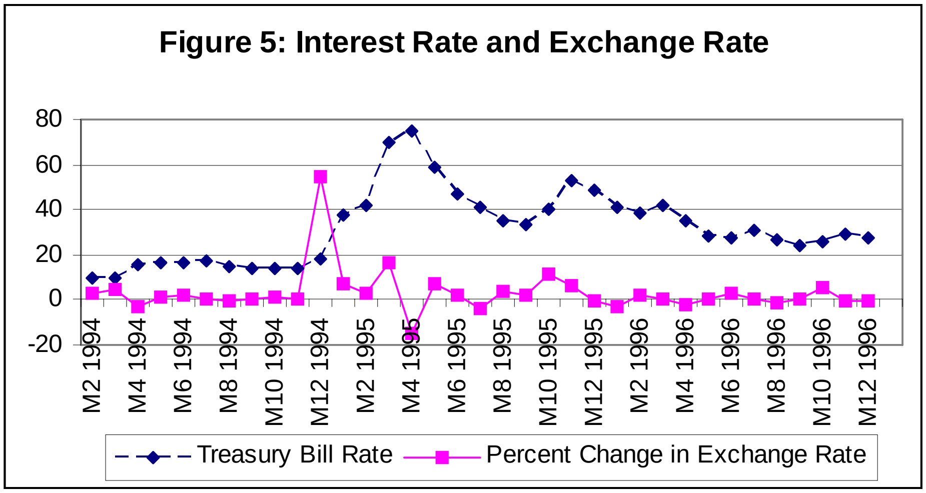
Compute δ_{t-1}using initial values for the debt and surplus and equation (13).
Compute $E_{t-1}\gamma_t$, based on equation (24) in the appendix.
Draw a fiscal shock, ν_t, from the uniform distribution.
Calculate γ_t using equation (15).
If $\gamma_t \leq 0$, update the surplus and debt using equations (6) and (4), and update δ_t.
If $\gamma_t > 0$ or $\delta_t < 0$ a crisis is called and the simulation ends.
If not, repeat up to twenty periods.
Repeat the twenty-year simulation 5000 times.

Table 2

Crisis Simulations: Argentina

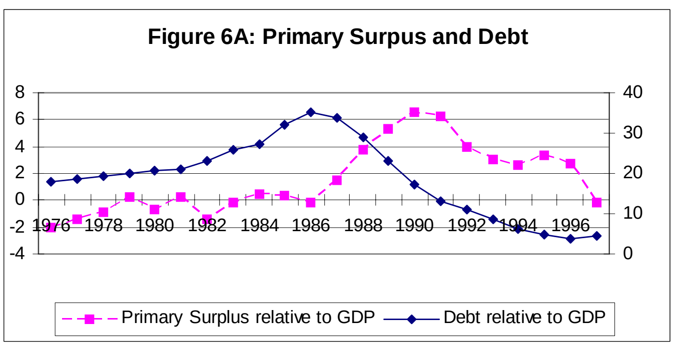
	b	**s**	$\boldsymbol{\alpha}$	$\hat{\mathbf{b}}$	**Pr Crisis (#yrs)**	**Mean time to Crisis**
1991	38.5	-0.5	0.61	75.0	0 (20)	–
1991	38.5	-0.5	0.13	62.5	69.5% (20)	18.0
1998	40.9	0.6	0.61	75	0.02% (20)	5.2
1998	40.9	0.6	0.13	62.5	60.8% (20)	6.5
1998	40.9	0.6	0.13	62.5	22.4% (3)	2
2000	50.9	0.4	0.61	75.0	0.6% (20)	17.1
2000	50.9	0.4	0.13	62.5	62.5% (3)	0.9

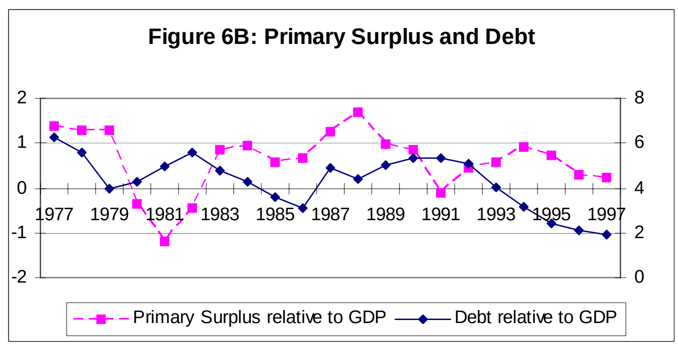
Mexico

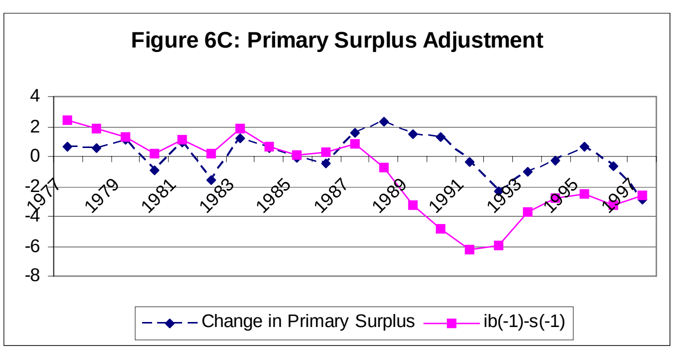


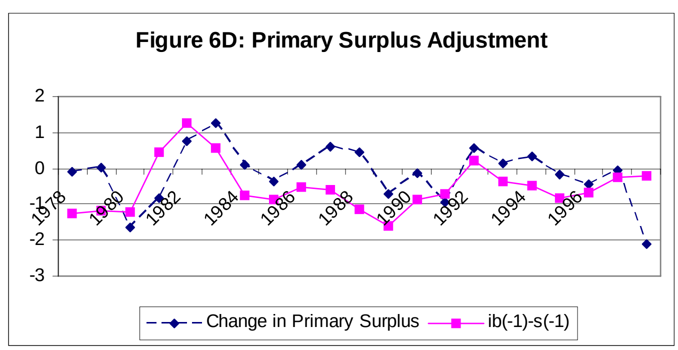
Thailand

Korea









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