

Novel Approaches to Coherency Conditions in LDV Models  
with an Application to Interactions between  
Financing Constraints  
and a Firm's Decision and Ability to Innovate

by

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Abstract

The paper discusses the key identification issue of *coherency conditions* in LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables. Two novel methods for establishing coherency conditions are presented, which have intuitive interpretations and are easy to implement and generalize. A major advantage of the new approaches is that they indicate how to achieve coherency in models traditionally classified as incoherent through the use of prior sign restrictions on model parameters. This allows us to develop estimation strategies based on Conditional MLE for simultaneous LDV models without imposing recursivity. Extensions of the analysis are given to simultaneous ordered probit models with multiple regions.

We also develop and summarize the results of a set of extensive Monte-Carlo experiments are used to evaluate the properties of the proposed Conditional MLE and the consequences of employing estimators that make overly restrictive coherency assumptions about the DGP. These experiments confirm very substantive improvements in terms of estimation Mean-Squared-Error by employing the CMLE developed in this paper.

We then employ our CMLE approach to obtain for the first time estimates of the reverse as well as direct interaction terms in LDV models with simultaneity that analyze the existence and impact of financing constraints as a possibly serious obstacle to innovation by firms. Direct measures of financing constraints are employed using survey data collected by the Banque de France and Eurostat, which overcomes the problems with the traditional approach of trying to deduce indirectly the existence and impact of financing constraints through the significance of firm wealth variables. The importance of using direct as opposed to indirect measures of financing constraints has been illustrated recently using a synthetic sample methodology and through the method of simulated moments). The econometric framework we employ in our study is the simultaneous bivariate probit with mutual endogeneity of direct indicators of financial constraints and innovation decisions by firms. Using the CMLE approach developed in the theoretical part of the paper, we quantify the

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interaction between financing constraints and a firm's decision and ability to innovate without forcing the econometric models to be recursive. Hence, we obtain direct as well as reverse interaction effects, leading us to conclude that binding financing constraints discourage innovation and at the same time innovative firms are more likely to face binding financing constraints. Finally, we establish a strong role for state-dependence in dynamic versions of our models.

**Keywords:** Limited Dependent Variable Models, Coherency Conditions, Joint Bivariate Probit Model, Linear Probability Models; Finance Constraints, Innovation, Econometric Coherency Conditions

**JEL Classifications:** C51, C52, C15

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# 1 Introduction

The paper discusses the major identification issue of *coherency conditions* in LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables. The econometric framework of LDV models with simultaneity is presented in Section 2. In the same section we explain the identification issue of *coherency* in such LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables using various illustrative models.

Conditions for coherency as discussed in the existing literature are reviewed and shown to be rather esoteric. Two novel methods for establishing coherency conditions are presented in Section 3, one based on a graphical characterization, the second through hypothetical Monte-Carlo DGP. The novel approaches have intuitive interpretations and are easy to implement and generalize. The constructive consequence of the new approaches is that they indicate how to achieve coherency in models traditionally classified as incoherent through the use of prior sign restrictions on model parameters. This allows us to develop estimation strategies in section 4 based on Conditional MLE for simultaneous LDV models without imposing recursivity. Thus one can obtain for the first time estimates of direct as well as reverse interaction effects in simultaneous LDV models, unlike in the existing literature where recursivity had to be assumed. Extensions to simultaneous ordered probit models with multiple regions and to panel LDV models are given in Section 5.

The proposed Conditional MLE methodology is evaluated through an extensive set of Monte-Carlo experiments described in Appendix 1. The key results are summarized in Section 6. These experiments allow us also to study the consequences of employing estimators that make overly restrictive coherency assumptions about the DGP. The findings confirm very substantive improvements in terms of estimation Mean-Squared-Error by employing the CMLE developed in this paper. In Section 7 we present an application of the econometric models of this paper to analyze and quantify interactions between financing constraints and firm innovation. Section 8 concludes.

## 2 The Econometric Problem of “Coherency” in LDV Models

In this section we present and study the fundamental identification issue of *coherency* of LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables. An LDV model was defined originally by [Gourieroux et al., 1980] to be “coherent” if it implies a valid function from the unobservables that drive the model to the observed limited dependent variables.

## 2.1 Defining Coherency using a Simultaneous LDV Model with Two Interactive Responses

We focus our discussion of the coherency problem in LDV models by using the Simultaneous LDV Model with Two Binary Responses. In this model, limited dependent variables  $y_1$  and  $y_2$  are jointly determined through filter functions  $\tau_1(\cdot)$  and  $\tau_2(\cdot)$  operating on latent variables  $y_1^*$  and  $y_2^*$  respectively:

$$y_{1it} = \tau_1(y_{1it}^* \equiv [h_1(x'_{1it}\beta_1, y_{2it}\gamma) + \epsilon_{1it}]) \quad (1)$$

$$y_{2it} = \tau_2(y_{2it}^* \equiv [h_2(x'_{2it}\beta_2, y_{1it}\delta) + \epsilon_{2it}]) \quad (2)$$

The (possibly non-linear) functions  $h_1(\cdot)$  and  $h_2(\cdot)$  are known up to parameter vectors  $\beta_1$  and  $\beta_2$  and the two interaction coefficients  $\gamma$  and  $\delta$ . The interaction terms  $y_{2it}\gamma$  and  $y_{1it}\delta$  appear in the respective latent variables  $y_{1it}^*$  and  $y_{2it}^*$ . The parameter vector to be estimated is  $\theta \equiv (\beta'_1, \beta'_2, \gamma, \delta, \sigma_1^2, \sigma_2^2, \rho)'$  where  $\rho \equiv correlation(\epsilon_{1it}, \epsilon_{2it})$ . In the most general case, the sample is a panel data set indexed by  $i = 1, \dots, N$  and  $t = 1, \dots, T$ .

The existing econometric literature has established as the typical coherency condition to be:  $\gamma \cdot \delta = 0$ , i.e., no reverse interaction terms are allowed among the two endogenous variables. This condition is sufficient for the joint distribution  $(y_{1it}, y_{2it}|x_1, x_2, \theta)$  to be well-specified is:  $\gamma \cdot \delta = 0$ . [Gourieroux et al., 1980] explain condition in terms of there being a valid function from  $(\epsilon_{1it}, \epsilon_{2it})$  to the observable endogenous variables  $(y_{1it}, y_{2it})$ . [Lewbel, 2007] establishes necessary and sufficient for coherency by approaching problem as requiring a valid reduced form system for  $(y_{1it}, y_{2it})$ . For example, if  $\delta = 0$  then the RF for  $y_{2it}$  is:

$$y_{2it} = \tau_2(h_2(x'_{2it}\beta_2) + \epsilon_{2it})$$

and hence the RF for  $y_{1it}$  is given by:

$$y_{1it} = \tau_1(h_1(x'_{1it}\beta_1, \gamma \cdot \tau_2(h_2(x'_{2it}\beta_2) + \epsilon_{2it})) + \epsilon_{1it})$$

## 2.2 General Explanation and Illustrative Applications

### 2.2.1 Application 1: Simultaneous Probit

The leading case we focus on here is the binary threshold crossing response model defined by:

$$\tau_j(z) \equiv \mathbf{1}(z > 0)$$

In terms of the two latent variables  $y_1^*$  and  $y_2^*$  and the observed binary indicators  $y_1$  and  $y_2$ , and suppressing the observation indices:

$$y_1 = \begin{cases} 1 & \text{if } y_1^* \equiv x_1\beta_1 + \gamma y_2 + \epsilon_1 > 0 \\ 0 & \text{if } y_1^* \equiv x_1\beta_1 + \gamma y_2 + \epsilon_1 \leq 0 \end{cases} \quad (3)$$

$$y_2 = \begin{cases} 1 & \text{if } y_2^* \equiv x_2\beta_2 + \delta y_1 + \epsilon_2 > 0 \\ 0 & \text{if } y_2^* \equiv x_2\beta_2 + \delta y_1 + \epsilon_2 \leq 0 \end{cases} \quad (4)$$

For a typical  $i$  observation, the probability  $Prob(y_{1it}, y_{2it}|X, \theta)$  is characterized by the constraints on the unobservables:

$$(a_1, a_2)' < (\epsilon_1, \epsilon_2)' < (b_1, b_2)'$$

through the configuration:

$y_{1it}$	$y_{2it}$	$a_1$	$b_1$	$a_2$	$b_2$
1	1	$-x_{1it}\beta_1 - \gamma$	$\infty$	$-x_{2it}\beta_2 - \delta$	$\infty$
1	0	$-x_{1it}\beta_1$	$\infty$	$-\infty$	$-x_{2it}\beta_2 - \delta$
0	1	$-\infty$	$-x_{1it}\beta_1 - \gamma$	$-x_{2it}\beta_2$	$\infty$
0	0	$-\infty$	$-x_{1it}\beta_1$	$-\infty$	$-x_{2it}\beta_2$

In this case,  $(y_1, y_2) \in \{(1, 1), (1, 0), (0, 1), (0, 0)\}$  such that:

$(y_{1it}, y_{2it})$	$y_{1it}^*$	$y_{2it}^*$
(1, 1)	$x'_{1it}\beta_1 + \gamma + \epsilon_{1it} > 0$	$x'_{2it}\beta_2 + \delta + \epsilon_{2it} > 0$
(1, 0)	$x'_{1it}\beta_1 + \epsilon_{1it} > 0$	$x'_{2it}\beta_2 + \delta + \epsilon_{2it} < 0$
(0, 1)	$x'_{1it}\beta_1 + \gamma + \epsilon_{1it} < 0$	$x'_{2it}\beta_2 + \epsilon_{2it} > 0$
(0, 0)	$x'_{1it}\beta_1 + \epsilon_{1it} < 0$	$x'_{2it}\beta_2 + \epsilon_{2it} < 0$

In general, in the absence of coherency conditions, there will be *overlaps* and/or *gaps* in the domain of  $(\epsilon_{1it} + x'_{1it}\beta_1, \epsilon_{2it} + x'_{2it}\beta_2)$ .

### 2.2.2 Application 2: Binary and Trinomial Ordered Model

Let us use a slightly more complicated simultaneous LDV model to analyze coherency, namely the *binary & trinomial ordered probit model* of [Hajivassiliou and Ioannides, 2007]. This model studies interactions between liquidity and employment constraints on individual households indexed by  $i$  at a given point in time indexed by  $t$ . The reason we select this model is because it can exhibit simultaneously *both* types of incoherency, namely overlaps and gaps in latent variable space. Define two latent dependent variables  $y_{1it}^*$  and  $y_{2it}^*$  and drop the  $it$  subscripts:

$$S = \begin{cases} 1 & \text{if } y_1^* > 0 \text{ (liquidity constraint binding),} \\ 0 & \text{if } y_1^* \leq 0 \text{ (liquidity constraint not binding).} \end{cases}$$

$$E = \begin{cases} -1 & \text{if } y_2^* \leq \lambda^- \text{ (overemployed)} \\ 0 & \text{if } \lambda^- \leq y_2^* < \lambda^+ \text{ (voluntarily employed)} \\ +1 & \text{if } \lambda^+ \leq y_2^* \text{ (under-/unemployed).} \end{cases}$$

$$y_1^* = \mathbf{1}(y_2^* < \lambda^-)\gamma_{11} + \mathbf{1}(\lambda^- < y_2^* < \lambda^+)\gamma_{12} + x'_1\beta_1 + \epsilon_1$$

$$y_2^* = \mathbf{1}(y_1^* > 0)\delta + x_2\beta_2 + \epsilon_2$$

Since  $(S, E)$  lie in  $\{0, 1\} \times \{-1, 0, 1\}$ , the 6 possible configurations may be enumerated as follows:

$S$	$E$	$y_1^*$	$y_2^*$
0	-1	$\gamma_{11} + x_1\beta_1 + \epsilon_1 < 0,$	$x_2\beta_2 + \epsilon_2 < \lambda^-$
0	0	$x_1\beta_1 + \epsilon_1 < 0,$	$\lambda^- < x_2\beta_2 + \epsilon_2 < \lambda^+$
0	+1	$\gamma_{12} + x_1\beta_1 + \epsilon_1 < 0,$	$\lambda^+ < x_2\beta_2 + \epsilon_2$
1	-1	$\gamma_{11} + x_1\beta_1 + \epsilon_1 > 0,$	$\delta + x_2\beta_2 + \epsilon_2 < \lambda^-$
1	0	$x_1\beta_1 + \epsilon_1 > 0,$	$\lambda^- < \delta + x_2\beta_2 + \epsilon_2 < \lambda^+$
1	+1	$\gamma_{12} + x_1\beta_1 + \epsilon_1 > 0,$	$\lambda^+ < \delta + x_2\beta_2 + \epsilon_2$

In terms of the unobservables, the probability of a  $(y_1, y_2)$  observed pair is equivalent to the probability<sup>2</sup>:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} < \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} < \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

where  $(\epsilon_1, \epsilon_2)' \sim N(0, \Sigma_\epsilon)$ , and  $a$  and  $b$  are given by:

$S$	$E$	$a_1$	$a_2$	$b_1$	$b_2$
0	-1	$-\infty$	$-\infty$	$-(\gamma_{11} + x_1\beta_1)$	$\lambda^- - x_2\beta_2$
0	0	$-\infty$	$\lambda^- - x_2\beta_2$	$-x_1\beta_1$	$\lambda^+ - x_2\beta_2$
0	+1	$-\infty$	$\lambda^+ - x_2\beta_2$	$-(\gamma_{12} + x_1\beta_1)$	$+\infty$
1	-1	$-(\gamma_{11} + x_1\beta_1)$	$-\infty$	$+\infty$	$\lambda^- - \delta - x_2\beta_2$
1	0	$-x_1\beta_1$	$\lambda^- - \delta - x_2\beta_2$	$+\infty$	$\lambda^+ - \delta - x_2\beta_2$
1	+1	$-(\gamma_{12} + x_1\beta_1)$	$\lambda^+ - \delta - x_2\beta_2$	$+\infty$	$+\infty$

Using traditional arguments, we obtain that a sufficient condition for coherency of the model is:

$$(\gamma_{11} + \gamma_{12})\delta = 0 \text{ and } \gamma_{11}\gamma_{12}\delta = 0.$$

- To verify this condition, suppose  $(S, E) = (0, 0)$ . This rules out  $(S, E) = (0, -1)$  because  $x_2\beta_2 + \epsilon_2 > \lambda^-$ , and rules out  $(S, E) = (1, 0)$  because  $x_1\beta_1 + \epsilon_1 < 0$ .
- But  $(1, -1)$  is not ruled out if the coherency conditions do not hold, since  $\gamma_{11}$  could be sufficiently negative and  $\delta$  sufficiently positive to imply the  $(1, -1)$  conditions.
- Similarly, the  $(1, 1)$  possibility cannot be ruled out in the absence of the coherency conditions, since  $\gamma_{12}$  and  $\delta$  can be sufficiently positive.
- Such logical inconsistencies are prevented if either (a)  $\delta = 0$  or (b)  $\gamma_{11}$  and  $\gamma_{12}$  are simultaneously 0.

Similar considerations can be employed to establish that the traditional coherency condition for the joint binary probit model (3)-(4) *while assuming no intertemporal endogeneity or dynamics* is:  $\gamma \cdot \delta = 0$ . This condition, of course, translates to the model being *recursive*. See [Maddala and Lee, 1976].

<sup>2</sup>The variance-covariance matrix captures the contemporaneous correlation between  $\epsilon_1$  and  $\epsilon_2$ . Given the binary nature of  $S$ ,  $\sigma_{11}$  is normalized to 1.

### 3 Extending the Traditional Approach to Coherency Conditions

The traditional approaches to the problem of coherency suffer from major difficulties: The first is that derivations of formal conditions using the traditional approach lack intuition. Second, they are practically impossible to generalize and verify in moderately more complicated LDV models, especially in cases where the models are allowed to contain intertemporal endogeneity of the type contained in [Falcetti and Tudela, 2007]. It is also very important to note that in case the joint binary probit model (3)-(4) is extended intertemporally, as for example in the dynamic panel probit model below in Section 5.2 and the empirical dynamic application in Section 7.6, the coherency condition is practically impossible to generalize and verify using the traditional analysis given in the previous paragraph. The third major difficulty is that in practice, non-triangular or reverse triangular cases are the most interesting from an economic point of view. Finally, the traditional approaches focus on establishing *sufficient* conditions for coherency, while our methods allow us to prove that they are *not necessary*.

To overcome the first two difficulties, alternative ways for establishing coherency are developed here, that are both intuitive and straightforward, as well as much more generalizable. In addition, our methods allow us to resolve the last two difficulties leading to estimation based on Conditional MLE for much more interesting practical applications. It is shown in the next Section how to establish coherency without recursiveness through the use of (a) endogeneity in terms of latent variables and/or (b) sign restrictions on model parameters. The fact that our novel approach for the first time eliminates the need to assume recursivity is quite important for the economic problem studied in the empirical application in Section 7 below. As we explain there, recursivity corresponds to the key identifying assumption that firm innovation does not affect financial distress directly ( $\delta = 0$ ). On a priori grounds, this assumption seems particularly dubious since firm innovation may lead to more profits and thus relax financial constraints (corresponding to  $\delta > 0$ ). An alternative possibility is that innovation may lead to higher investment in intangible assets thus reinforcing binding financial constraints (corresponding to  $\delta < 0$ ). Both possibilities violate the traditional coherency condition.<sup>3</sup>

#### 3.1 Novel Approach 1: Graphical

Let us illustrate the first approach using the Liquidity-Employment constraints application of Hajivassiliou and Ioannides (op.cit.). This graphical approach was first included in the LSE working paper Hajivassiliou (2002) and was presented at the CRETE Conference in Syros in 2003. It should also be noted that our graphical approach we present here is related to that of [Tamer, 2003] who studied the problem of coherency in bivariate discrete models for games with multiple equilibria.

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<sup>3</sup>Note that throughout we expect  $\gamma < 0$ , i.e., the higher the probability that a firm faces a binding financial constraint, the less likely it is that it is able to innovate. So the two possibilities translate to: (a)  $\gamma < 0$ ,  $\delta > 0$  and (b)  $\gamma < 0$ ,  $\delta < 0$ .

Figure 1 overleaf gives the 6 possible regimes  $(S \times E) = \{1, 0\} \times \{-1, 0, 1\}$  in terms of the two latent variables  $y_1^*$  and  $y_2^*$  and the possible configurations in terms of parameters  $\bar{\lambda}$ ,  $\lambda$ ,  $\delta$ ,  $\gamma_{11}$ , and  $\gamma_{12}$ .  $y_1^*$  is on the horizontal axis and  $y_2^*$  on the vertical. The figure makes clear the role of the coherency condition (a)  $\delta = 0$  or (b)  $\gamma_{11} = \gamma_{12} = 0$ : in general, regions  $R2$  and  $R6$  exhibit double-counting (cross-hatched area), as well as a white rectangle remains which makes the six regions not mutually exhaustive. These two logical incoherencies disappear when either  $\delta = 0$  and/or  $\gamma_{11} = \gamma_{12} = 0$  hold.

We develop further our graphical approach in Section 4 below, and use it to highlight the fundamental distinction between two types of incoherency, the first corresponding to overlap regions in latent variables space, while the second to empty regions. We explain there that incoherencies of the latter type can be overcome through additional prior sign restrictions on model parameters through the use of Conditional MLE.

### 3.2 Novel approach 2: DGP From First Principles

Despite the usefulness of the graphical approach of the previous section to LDV problems with two latent variables, the method is very unwieldy or inapplicable to higher dimensional cases. To cover such problems, we develop a second approach to incoherency, which consists of designing a data-generating algorithm (on a computer or hypothetical) to simulate random draws from an LDV model's structure. Again let us use the Liquidity-Employment Constraints application of Hajivassiliou and Ioannides (op.cit.) to illustrate the method. We draw  $\epsilon_1$  and  $\epsilon_2$  under the joint bivariate normal distribution with zero mean vector and variance-covariance matrix  $\Sigma_\epsilon$ , and given  $x_1\beta_1$  and  $x_2\beta_2$  attempt to generate  $y_1^*$  and  $y_2^*$ . This is possible provided the coherency condition holds: If (a)  $\delta = 0$ , then latent  $y_2^*$  can be drawn, then ldv  $y_2$ , which together with  $\epsilon_1$  and  $x_1\beta_1$  determines the rhs of  $y_1^*$ , thus allowing  $y_1$  to be drawn. Similarly, if (b)  $\gamma_{11} = \gamma_{12} = 0$ , then  $y_1^*$  can be drawn from the first equation based on  $\epsilon_1$  and  $x_1\beta_1$ , which determines  $y_1$ , thus giving  $y_2^*$  and hence  $y_2$ . It is not obvious, however, whether such data generation can be achieved in case the coherency condition does not hold. This approach is related to the [Gourieroux et al., 1980] condition that a function exist from  $\epsilon_1, \epsilon_2$  to  $y_1, y_2$ .

As we will show in section 5.2 below, the approach extends naturally to cases with intertemporal endogeneities in panel LDV models, and can be used to prove the coherency of the classic multiperiod panel probit with state dependence ([Heckman, 1981b]), as well as the general versions of illustrative models [Section 7.5] and [Section 7.6] with explicit dynamic effects.

## 4 Identification Under Additional Prior Sign Restrictions

The graphical approach we developed in the previous section highlights two distinct cases of incoherency: the first type of incoherency corresponds to regions of the

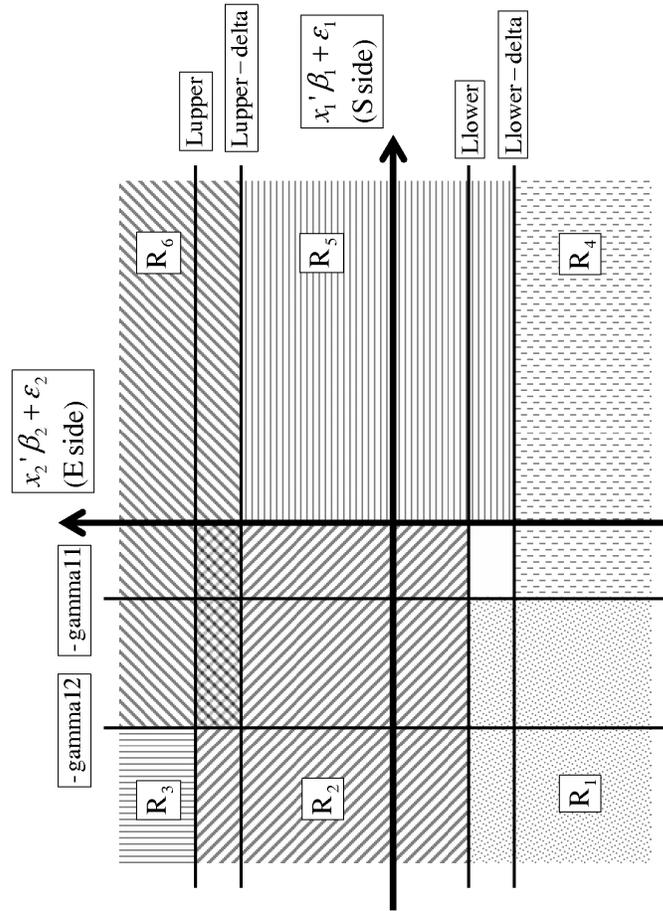


Figure 1: Regions in latent variables space that define the observed dependent qualitative variables Liquidity (binary) and Employment (trinomial) Constraints. Consequently there are six possible regimes. We see two regions of incoherency, one cross-hatched, the other empty. Both regions disappear under the coherency condition that  $\Delta=0$  and/or  $\Gamma_{11}=\Gamma_{12}=0$ .

Figure 1: Coherency of Binary+Trinomial Model: [Liquidity and Employment Constraints]

observed endogenous variables of the model being *overlapping*, while the second to regions that are *empty*. We show that empty region incoherency can be overcome through conditional maximum likelihood (CMLE) of truncating the LDVs to lie outside the incoherency regions.<sup>4</sup>

Our CMLE approach we propose here can also be motivated through the DGP approach for establishing coherency that we discussed in the previous subsection. In that case, we need to consider DGPs truncated to lie on a specific region of the latent variables space. A specific method for achieving this is given in technical Appendix 1 below.

It is useful to highlight here the similarities and differences to the analysis in [Tamer, 2003], who also used a graphical approach to resolve an incomplete simultaneous discrete response model for a homogeneous two-agent discrete game of entry. Since the two rival firms in his setting were assumed identical, any incoherency arising was necessarily of the indeterminate type — see our two subcases 4.1 and 4.2 below, where the interaction terms  $\gamma$  and  $\delta$  are of the same sign. Consequently, the possibility of the interaction terms being of opposite sign was not under focus in his analysis and hence the applicability of Conditional MLE to resolve those cases was not considered. explain his model. why didn't consider opposite signs. It is also useful to note that our approach for establishing coherency through the use of prior sign restrictions developed here is related to the recent approach by [Uhlig, 2005] for VAR identification under prior sign restrictions on impulse response functions.<sup>5</sup> [Dagenais, 1997] also makes a distinction between alternative types of incoherency regions.<sup>6</sup>

We illustrate the Conditional MLE approach for establishing Coherency through prior sign restrictions by using the joint binary probit model:<sup>7</sup>

$$I = \begin{cases} 1 & \text{if } I^* \equiv x^I \beta^I + \gamma F + \epsilon^I > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F = \begin{cases} 1 & \text{if } F^* \equiv x^F \beta^F + \delta I + \epsilon^F > 0 \\ 0 & \text{otherwise} \end{cases}$$

Obviously, there exist **four cases** based on signs of  $\gamma, \delta$ :

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<sup>4</sup>We also explain below that overlapping region incoherency *cannot* be transformed into empty region incoherency by redefining one of the observed binary LDVs to its complement.

<sup>5</sup>We are indebted to Alain Trognon for pointing out the potential of parameter sign restrictions overcoming incoherency of the “empty region” type, and to Hashem Pesaran for bringing to our attention Uhlig’s work on sign identification.

<sup>6</sup>Unfortunately his work remains incomplete and unpublished due to his untimely death.

<sup>7</sup>For the first equation,  $I^*$  is used for the latent and  $I$  for the observed LDV as a mnemonic to the *Innovation* side of the model of Section 7 below. Similarly, for the second equation we use  $F^*$  and  $F$  as a mnemonic to *Financing Constraints*.

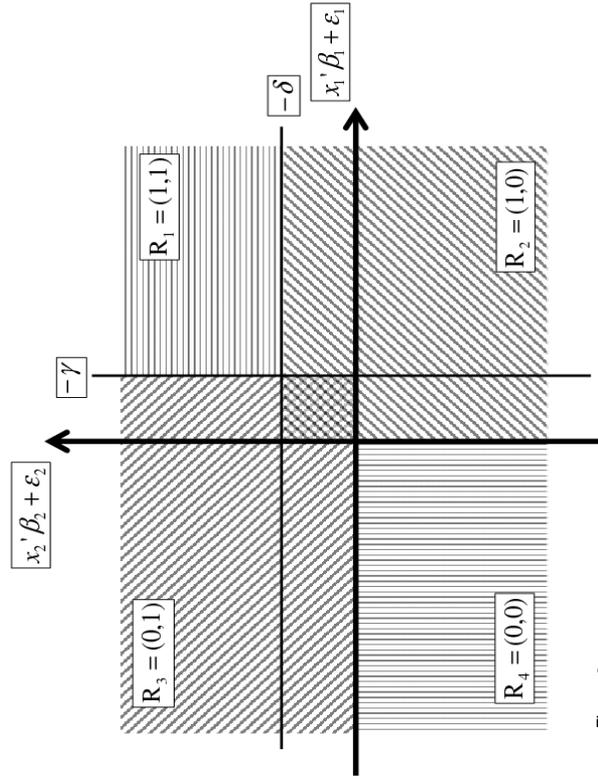


Figure 2:  
 Innovation and Finance Constraint bivariate binary model in latent variables space.  
 Four implied regimes. Case 1:  $\gamma > 0, \delta > 0$ . Region of incoherency is of the  
 overlap type (cross-hatched).

Figure 2: Case 1:  $\gamma > 0, \delta > 0$

#### 4.1 Case 1: $\gamma > 0, \delta > 0$ — overlapping regions, incoherency

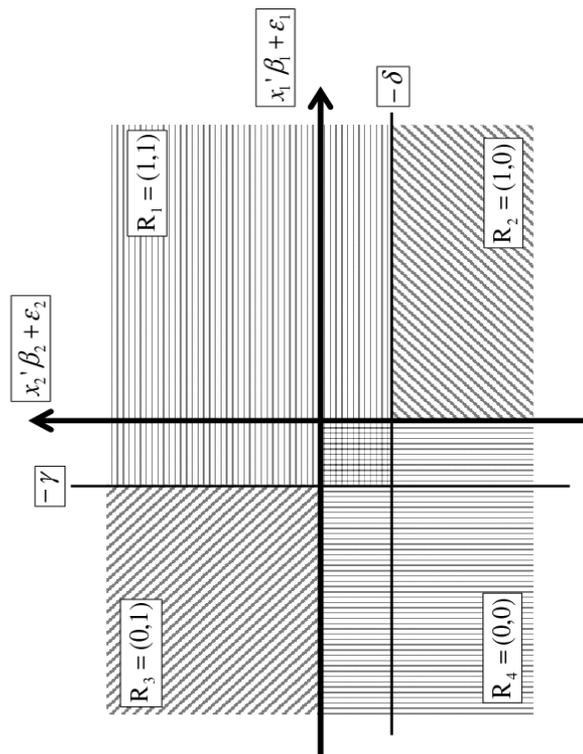


Figure 3: Innovation and Finance Constraint bivariate binary model in latent variables space. Four implied regimes. Case 2:  $\gamma < 0, \delta < 0$ . Region of incoherency is of the overlap type (cross-hatched).

Figure 3: Case 2:  $\gamma < 0, \delta < 0$

#### 4.2 Case 2: $\gamma < 0, \delta < 0$ — overlapping regions, incoherency

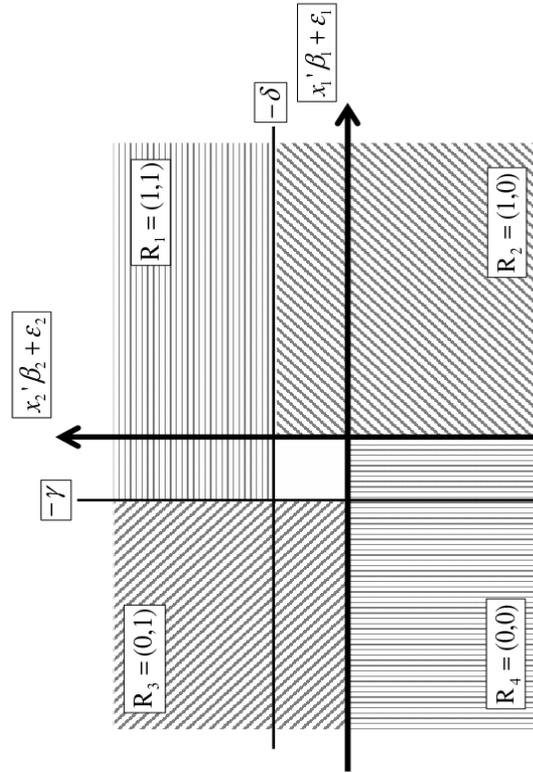


Figure 4: Innovation and Finance Constraint bivariate binary model in latent variables space. Four implied regimes. Case 3:  $\gamma > 0, \delta < 0$ . Region of incoherency is of the empty region type. Hence, Conditional MLE achieves coherency by ruling out the event:  $[-\gamma < \epsilon_1 + x_1\beta_1 < 0, 0 < \epsilon_2 + x_2\beta_2 < -\delta]$

Figure 4: Case 3:  $\gamma > 0, \delta < 0$

### 4.3 Case 3: $\gamma > 0, \delta < 0$ — empty regions, coherency through conditioning

For this case, coherency can be achieved by conditioning to lie outside the “empty” region of figure 5, which has conditioning probability:

$$1 - Prob(-\gamma < \epsilon_1 + x_1\beta_1 < 0, 0 < \epsilon_2 + x_2\beta_2 < -\delta)$$

The estimation method that implements this is Truncated MLE

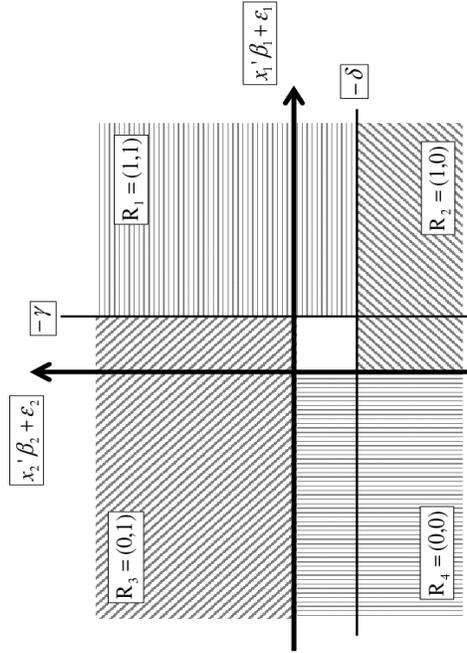


Figure 5:  
 Innovation and Finance Constraint bivariate binary model in latent variables space.  
 Four implied regimes. Case 4:  $\gamma < 0, \delta > 0$ . Region of incoherency is of the empty region  
 type. Hence, Conditional MLE achieves coherency by ruling out the event:  
 $[0 < \epsilon_1 + x_1\beta_1 < -\gamma, \delta < \epsilon_2 + x_2\beta_2 < 0]$

Figure 5: Case 4:  $\gamma < 0, \delta > 0$

#### 4.4 Case 4: $\gamma < 0, \delta > 0$ — empty regions, coherency through conditioning

For this case also, coherency is achieved by conditioning to lie outside the “empty” region of figure 4. The conditioning probability is:

$$1 - \text{Prob}(0 < \epsilon_1 + x_1\beta_1 < -\gamma, \delta < \epsilon_2 + x_2\beta_2 < 0)$$

and the appropriate estimation method is Truncated MLE.

## 4.5 To Show that Models with Overlapping Regions Remain Incoherent Irrespective of LDV Definitions

We have shown that in general, in the absence of coherency conditions, there will be *overlaps* and/or *gaps* in the domain of  $(\epsilon_1 + x'_1\beta_1, \epsilon_2 + x'_2\beta_2)$ . At this point, a researcher might be tempted to propose that the incoherency cases with overlapping regions (Cases 1 and 2 above) may be overcome by redefining one of the two limited dependent variables to their complement. According to this reasoning, since the incoherency is caused in these cases because  $\gamma$  and  $\delta$  are of the same sign, and since changing  $y_2$ , say, to its complement  $y_2^N \equiv (1 - y_2)$  would result in  $\delta^N \equiv -\delta$ , then coherency would be achieved since then  $\gamma \cdot \delta^N < 0$ .

Such reasoning would be incorrect, however. We analyze here this idea and show that such a redefinition would *maintain* the overlapping-region incoherency. This is because the  $y_2^N \equiv (1 - y_2)$  redefinition would also switch the sign of  $\gamma$  and hence  $\gamma^N \cdot \delta^N > 0$  just as  $\gamma \cdot \delta > 0$ .

Let us return to the bivariate binomial probit model in terms of the two latent variables  $I^*$  and  $FC^*$  and the observed binary indicators  $I$  and  $FC$ , and suppressing the observation index:

$$I = \begin{cases} 1 & \text{if } I^* \equiv x_1\beta_1 + \gamma FC + \epsilon_1 > 0 \\ 0 & \text{if } I^* \equiv x_1\beta_1 + \gamma FC + \epsilon_1 \leq 0 \end{cases} \quad (5)$$

$$FC = \begin{cases} 1 & \text{if } FC^* \equiv x_2\beta_2 + \delta I + \epsilon_2 > 0 \\ 0 & \text{if } FC^* \equiv x_2\beta_2 + \delta I + \epsilon_2 \leq 0 \end{cases} \quad (6)$$

Suppose we have incoherency because we believe  $\gamma > 0$  (in the Hajivassiliou-Savignac study corresponding to binding  $FC$ s cause increasing chance of innovation  $I$ ) and that  $\delta > 0$  (firms who have high  $I$  i.e., innovate raise the chance the banks will refuse them a loan so high  $FC$ ). So  $\gamma \cdot \delta > 0$ . This is Case 1 analyzed in subsection 4.1 as represented by Figure 2, and corresponding to the constraints on the unobservables:

$$(a^1, a^2)' < (\epsilon^1, \epsilon^2)' < (b^1, b^2)'$$

such that:

$I$	$FC$	$a^1$	$b^1$	$a^2$	$b^2$	Shading	Region
1	1	$-x_1\beta_1 - \gamma$	$\infty$	$-x_2\beta_2 - \delta$	$\infty$	horizontal	R1
1	0	$-x_1\beta_1$	$\infty$	$-\infty$	$-x_2\beta_2 - \delta$	swne	R2
0	1	$-\infty$	$-x_1\beta_1 - \gamma$	$-x_2\beta_2$	$\infty$	nwse	R3
0	0	$-\infty$	$-x_1\beta_1$	$-\infty$	$-x_2\beta_2$	vertical	R4

Now consider the transformed model with  $NFC$  **instead of**  $FC$ . This transformation still gives an overlapping region in the transformed variables, and hence corresponds to an incoherent model. To see this, proceed as follows:

In terms of the two latent variables  $I^*$  and  $NFC^* = -FC^*$  and the observed binary indicators  $I$  and  $NFC = 1 - FC$ , and suppressing the observation index:

$$I = \begin{cases} 1 & \text{if } I^* \equiv x_1\beta_1 + \gamma^N NFC + \epsilon_1 > 0 \\ 0 & \text{if } I^* \equiv x_1\beta_1 + \gamma^N NFC + \epsilon_1 \leq 0 \end{cases} \quad (7)$$

$$NFC = \begin{cases} 1 & \text{if } NFC^* \equiv x_2\beta_2^N + \delta^N I + \epsilon_2^N > 0 \\ 0 & \text{if } NFC^* \equiv x_2\beta_2^N + \delta^N I + \epsilon_2^N \leq 0 \end{cases} \quad (8)$$

Given this transformation, we expect that  $\gamma^N < 0$  (high  $NFC$  means not very binding constraints so cause dampening of  $I$ ) and that  $\delta^N < 0$  (firms who have high  $I$  i.e., innovate raise the chance the banks will refuse them a loan so low  $NFC$ ). So  $\gamma^N \cdot \delta^N > 0$ . See illustration 6.

For a typical  $i$  observation, the probability  $Prob(y_{1i}, y_{2i}|X, \theta)$  is characterized by the constraints on the unobservables:

$$(a^1, a^2)' < (\epsilon_1, \epsilon_2^N)' < (b^1, b^2)'$$

through the configuration:

$I$	$NFC$	$a^1$	$b^1$	$a^2$	$b^2$	Shading	Region
1	0	$-x_1\beta_1$	$\infty$	$-\infty$	$-x_2^N\beta_2 - \delta^N$	horizontal	R1
1	1	$-x_1\beta_1 - \gamma^N$	$\infty$	$-x_2\beta_2^N - \delta^N$	$\infty$	swne	R2
0	0	$-\infty$	$-x_1\beta_1$	$-\infty$	$-x_2^N\beta_2$	nwse	R3
0	1	$-\infty$	$-x_1\beta_1 - \gamma^N$	$-x_2^N\beta_2$	$\infty$	vertical	R4

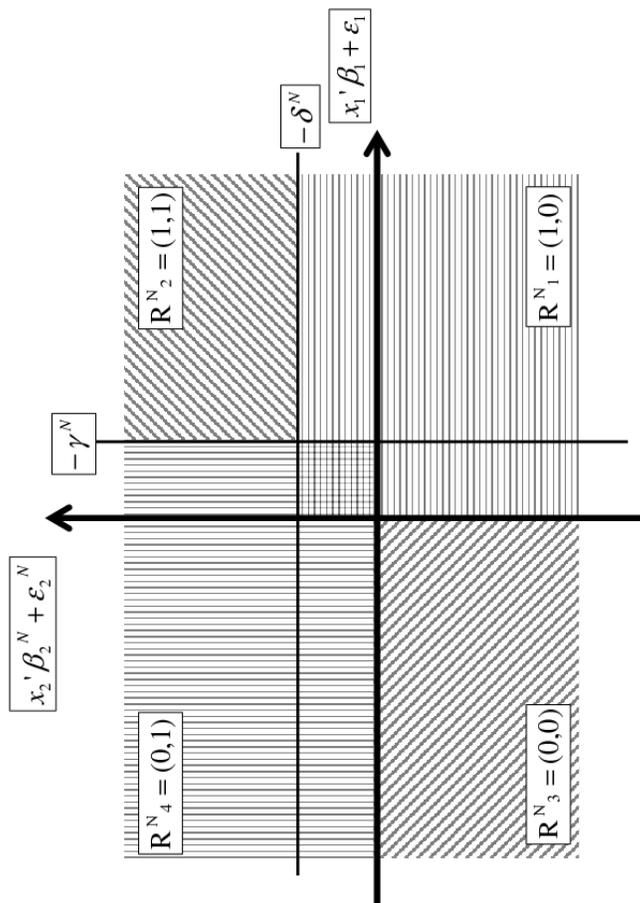


Figure 6:  $\gamma^N < 0, \delta^N < 0$

Figure 6: Innovation and Finance Constraint bivariate binary model in latent variables space. Situation identical to Figure 1 (Case 3) but with Innovation negated to its complement (1-Innovation). Both  $\gamma^N$  and  $\delta^N$  switch sign by this redefinition, and hence:  $\gamma^N < 0, \delta^N < 0$ . Consequently, the model remains incoherent with an overlapping (cross-hatched) region.

## 4.6 Efficient Estimation through Conditional Maximum Likelihood

The optimal parametric estimation approach for the models with empty region incoherency (Cases 3 and 4 above) will be *truncated conditional maximum likelihood*, employing the appropriate likelihood contributions that characterize correctly the necessary conditioning that ensures that the LDVs stay out of the empty region of incoherency. For example, assuming independence across observations  $i = 1, \dots, N$ , the likelihood contribution in Case 3 will be:

$$l_i = \text{Prob}(\epsilon_1, \epsilon_2 : I = 1(I^* > 0) \ \& \ F = 1(F^* > 0)) / (1 - \text{Prob}(-\gamma < \epsilon_1 + x_1\beta_1 < 0, 0 < \epsilon_2 + x_2\beta_2 < -\delta))$$

while for Case 4:

$$l_i = \text{Prob}(\epsilon_1, \epsilon_2 : I = 1(I^* > 0) \ \& \ F = 1(F^* > 0)) / (1 - \text{Prob}(0 < \epsilon_1 + x_1\beta_1 < -\gamma, \delta < \epsilon_2 + x_2\beta_2 < 0))$$

These likelihood contributions make it clear why approaches that ignore the coherency issue are inconsistent in general: the inconsistency would arise because the conditioning probability expressions in the denominator are functions of the underlying parameters and data, and hence affect critically the evaluation of the correct likelihood function.<sup>8</sup>

It should be noted also that Cases 1 and 2 may be handled in an analogous fashion *provided it is assumed first* that the Data Generating Process that overcomes the overlapping-regions incoherency is one where  $(\epsilon_{1i}, \epsilon_{2i})$  are drawn from an unrestricted bivariate normal distribution and then any draws falling into the overlap region are rejected. To find the correct likelihood contributions in these two cases, note that:

$$p_{11}^* + p_{10}^* + p_{01}^* + p_{00}^* = S > 1$$

where  $S - 1 \equiv d$ , the probability of the overlap region. In Case 1, the overlap occurs between regions (1, 1) and (0, 0), while for Case 2 between regions (1, 0) and (0, 1). Consequently, assuming an Accept/Reject DGP out of the overlap region, the

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<sup>8</sup>For completeness, we consider the case of Latent Variable Endogeneity. Let us modify the two-equation LDV model (1)-(2) to make the interaction terms be the *latent* variables instead of the *limited* counterparts:

$$\begin{aligned} y_{1it} &= \tau_1 (y_{1it}^* \equiv [h_1(x'_{1it}\beta_1, y_{2it}^*\gamma) + \epsilon_{1it}]) \\ y_{2it} &= \tau_2 (y_{2it}^* \equiv [h_2(x'_{2it}\beta_2, y_{1it}^*\delta) + \epsilon_{2it}]) \end{aligned}$$

Then:

$$\begin{aligned} y_1^* &= x_1\beta_1 + y_2^*\gamma + \epsilon_1 \\ y_2^* &= x_2\beta_2 + y_1^*\delta + \epsilon_2 \end{aligned}$$

and

$$\begin{aligned} y_1^* &= x_1\beta_1 + \gamma \cdot [x_2\beta_2 + y_1^*\delta + \epsilon_2] + \epsilon_1 \\ y_2^* &= x_2\beta_2 + \delta \cdot [x_1\beta_1 + y_2^*\gamma + \epsilon_1] + \epsilon_2 \end{aligned}$$

Hence  $y_1^* = RF_1$  and  $y_2^* = RF_2$ , allowing us to obtain  $y_1 = \tau(RF_1)$  and  $y_2 = \tau(RF_2)$ . We thus see that it is considerably more straightforward to establish coherency identification of LDV models with latent variable interactions as opposed to limited variable interactions.

likelihood contribution for Case 1 is:

$$l_i = \begin{cases} p_{11} = (p_{11}^* - d)/(2 - S) \\ p_{10} = p_{10}^*/(2 - S) \\ p_{01} = p_{01}^*/(2 - S) \\ p_{00} = (p_{00}^* - d)/(2 - S) \end{cases}$$

while for Case 2:

$$l_i = \begin{cases} p_{11} = p_{11}^*/(2 - S) \\ p_{10} = (p_{10}^* - d)/(2 - S) \\ p_{01} = (p_{01}^* - d)/(2 - S) \\ p_{00} = p_{00}^*/(2 - S) \end{cases}$$

## 5 Extensions

### 5.1 Bivariate Multinomial Ordered Probit Cases

We now discuss how to extend our analysis to the case of two simultaneous (bivariate) *ordered probit equations with multiple regions*.<sup>9</sup> Suppose we have a model given by

$$\begin{aligned} y_1^* &= \beta_1' x_1 + \delta_{y_2} + \epsilon_1 \\ y_2^* &= \beta_2' x_2 + \delta_{y_1} + \epsilon_2 \\ y_1 &= I_1(y_1^*) \\ y_2 &= I_2(y_2^*) \end{aligned}$$

where we define

$$\begin{aligned} I_1(y_1^*) &\in \{1, 2, \dots, n_1\} \\ I_2(y_2^*) &\in \{1, 2, \dots, n_2\} \\ I_1(y_1^*) &= \max\{i | y_1^* < s_{1i}\} \\ I_2(y_2^*) &= \max\{i | y_2^* < s_{2i}\} \end{aligned}$$

The sets  $\{s_{1i}\}$  and  $\{s_{2i}\}$  are  $n_1 - 1$  and  $n_2 - 1$  increasing transition values, and  $s_{1n_1} = s_{2n_2} = \infty$ , so that all very large values get mapped to the highest category. Then

$$\begin{aligned} \delta_{y_1} &\in \{\delta_{11}, \delta_{12}, \dots, \delta_{1n_2}\} \\ \delta_{y_2} &\in \{\delta_{21}, \delta_{22}, \dots, \delta_{2n_1}\} \end{aligned}$$

are interaction terms that take one of  $n_2$  and  $n_1$  discrete values depending on  $y_2$  and  $y_1$  respectively. The error terms,  $\epsilon_1$  and  $\epsilon_2$ , are assumed to be normally distributed

<sup>9</sup>The research assistance of Ryan Giordano has been especially helpful for this section.

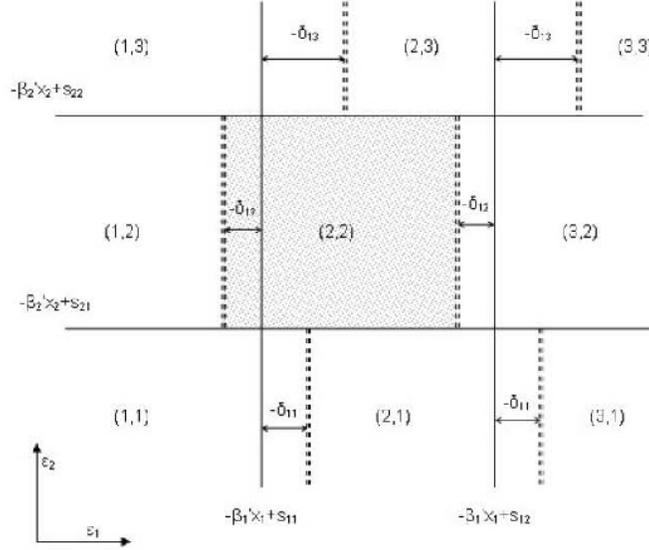


Figure 7: Coherency of Joint Ordered Response Models

conditional on lying outside of incoherent regions – that is, regions in which there is not a single, unambiguous pair  $y_1$  and  $y_2$  that corresponds to them.

The bivariate binary probit is a subset of this case, with  $n_1 = n_2 = 2$ ,  $s_{11} = s_{21} = 0$ ,  $\delta_{11} = \gamma$ ,  $\delta_{21} = \delta$ , using our usual notation.

As an illustration, consider the following figure. Here,  $n_1 = n_2 = 3$ , and all the  $\delta_{2i} = 0$  so that  $y_1$  does not affect  $y_2$ . As in the binary probit, the effect of the interaction terms are to shift the boundaries in  $\epsilon$ 's domain that map to particular outcomes for  $y_1$  and  $y_2$ . For example, the shaded area corresponds to  $y_1 = 2$ ,  $y_2 = 2$ .

In this picture, there are no incoherent regions, since the system is triangular. To proceed to determine the regions of incoherency in the non-triangular case, for simplicity we will assume that the interaction terms are small enough so that  $s_{1(i+1)} - s_{1i} > \delta_{2(j+1)} - \delta_{2j}$  for all  $i$  and  $j$ , and that a similar condition holds for the transition values and interaction terms of  $y_2$ . That is to say, the interaction terms are small relative to the threshold values. If this is not the case, counting overlaps becomes more complicated.

If this condition holds, then each intersection of threshold values becomes analogous to the binary bivariate probit case, except that neighbouring interaction terms now determine whether there is an empty region or overlap. For example, consider the following situation. Here,

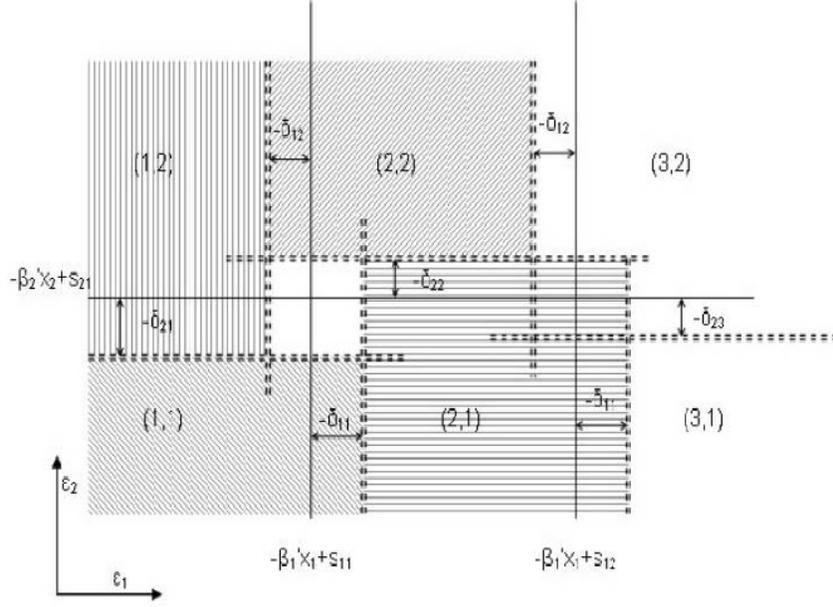


Figure 8: Coherency of Joint Ordered Response Models II

$$\begin{aligned}
 n_1 &= 3 \\
 n_2 &= 2 \\
 \delta_{11} &< 0 \\
 \delta_{12} &> 0 \\
 \delta_{21} &> 0 \\
 \delta_{22} &< 0 \\
 \delta_{23} &> 0
 \end{aligned}$$

When we draw in the shaded regions for the first corner, it is evident that there is an empty region. This is because  $\delta_{12} - \delta_{11} > 0$  has the same a different sign from  $\delta_{22} - \delta_{21} < 0$ . Indeed, if we were to take

$$\begin{aligned}
 z_1 &= -\beta_1'x_1 - \delta_{11}z_2 &= -\beta_2'x_2 - \delta_{21}\gamma_1 = \delta_{12} - \delta_{11} \\
 \gamma_2 &= \delta_{22} - \delta_{21}
 \end{aligned}$$

then the situation would be identical to the bivariate binary probit with the role of the exogenous variables played by  $z_1$  and  $z_2$ , and the relevant interaction terms

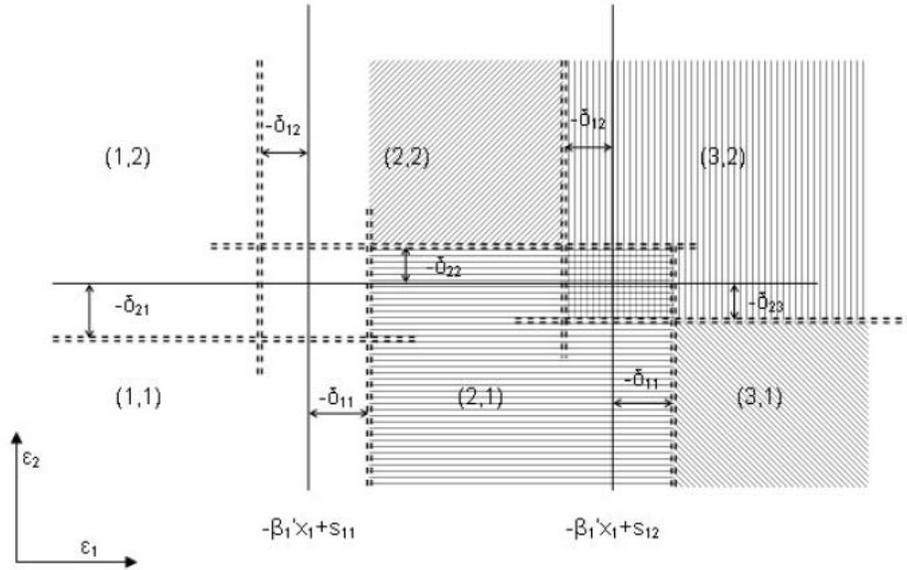


Figure 9: coherency of joint ordered Response Models III

being  $\gamma_1$  and  $\gamma_2$ . Here, for similar reasons, we will see an overlap at the second intersection:

Having observed this, it is easy to define a procedure to calculate the overall likelihood with coherency incorporated. For a particular outcome, one first calculates its incoherent probability (the probability of  $(\epsilon_1, \epsilon_2)$  landing in its bounding box). Then one checks each of its corners for overlaps with neighbouring regions, subtracting the probability of any overlap regions. Finally, one needs to divide by the total probability of a coherent draw, which equals one minus the sum of the probabilities of all the empty and overlapping regions.

We have investigated the theoretical probabilities for the model made coherent in this fashion, and compared them to the actual frequency probabilities from the postulated accept-reject DGP described in the first paragraph of this section. The two sets of probabilities generated in these investigations matched one another very satisfactorily.

## 5.2 Establishing the Coherency of Panel LDV Models with Intertemporal Endogeneities using DGP Approach

Extending the analysis to a panel data set, [Hajivassiliou, 2007] explains how the probability of a pair  $(S_{it}, E_{it})$  in subsection 2.2.2 and a pair  $(y_{1it}, y_{2it})$  in subsection 2.2.1, can be represented in terms of the linear inequality:

$$(a_1, a_2)' < (\epsilon_1, \epsilon_2)' < (b_1, b_2)'$$

where the error vector has a flexible autocorrelation structure. For example, one-factor random effect assumptions will imply an equicorrelated block structure on  $\Sigma_\epsilon$ , while our most general assumption of one-factor random effects *combined with* an AR(1) process for each error implies that  $\Sigma_\epsilon$  combines equicorrelated and Toeplitz-matrix features. Consequently, the approach incorporates fully (a) the contemporaneous correlations in  $\epsilon_{it}$ , (b) the one-factor plus AR(1) serial correlations in  $\epsilon_i$ , and (c) the dependency of  $S_{it}$  on  $E_{it}$  and vice versa. The coherency issue expands naturally to the panel sequence of data, by thinking of each (correlated) time-period for a given individual  $i$  as a **distinct probit equation** and then dealing with the independent cross-section of equations across individuals. Details of the analysis can be found in Hajivassiliou (op.cit.).

Our hypothetical DGP method presented in Subsection 3.2 for establishing coherency is now applied to the canonical panel data Probit model with state-dependence, first analyzed by Heckman (op.cit.). Let us begin with the simplified case of the initial condition being exogenous:

$$y_{iT} = \mathbf{1}(\lambda y_{i,T-1} + x_{iT}\beta + \epsilon_{iT} > 0) \quad (9)$$

$$y_{i,T-1} = \mathbf{1}(\lambda y_{i,T-2} + x_{i,T-1}\beta + \epsilon_{i,T-1} > 0) \quad (10)$$

$$\vdots \quad (11)$$

$$y_{i2} = \mathbf{1}(\lambda y_{i1} + x_{i2}\beta + \epsilon_{i2} > 0) \quad (12)$$

$$y_{i1} = \mathbf{exogenous} \quad (13)$$

Let  $\Sigma \equiv VCov(\epsilon_{iT}, \dots, \epsilon_{i1}, u_{i1})$ . Suppose first the  $\epsilon_{it}$  has the one-factor (equicorrelated) error components structure  $\epsilon_{it} = \alpha_i + \nu_{it}$ . Conditional on  $\alpha_i$ , these  $T - 1$  equations are independent (since they only depend on the i.i.d.  $\nu_{it}$ s. Hence draw an  $\alpha_i$  and an independent  $\nu_{i2}$ . Then use the exogenous  $y_{i1}$  outcome to generate  $y_{i2}$ . This completes equation 12 which allows to move recursively to generating  $y_{i3}$ , then  $y_{i4}$ , etc. until  $y_{iT}$  is generated. This establishes the coherency of the model.

Now consider the more general case when  $y_{i1}$  cannot be assumed as exogenous. We then supplement the system with an initial condition equation:

$$y_{i1} = \mathbf{1}(x_{i1}\xi_1 + \dots + x_{iT}\xi_T + u_{i1} > 0)$$

The following remarks are in order: First note that equation 1 is a generalization of the [Barghava and Sargan, 1982] approach. Second, one-factor random effect

assumptions will imply an equicorrelated block structure on the top left  $T - 1 \times T - 1$  block of  $\Sigma$ , while more general assumptions of one-factor random effects *combined with* an AR(1) or ARMA(p,q) processes for each  $\epsilon_{it}$  error implies that  $\Sigma$  combines equicorrelated and Toeplitz-matrix parts. The last row and last column of  $\Sigma$  giving the variance of  $u_{1i}$  and its covariances with all  $\epsilon_{it}$  allow the flexibility stipulated by [Heckman, 1981a]. Define the Cholesky lower triangular times upper triangular factorization of  $\Sigma = CC'$ . Given the assumed normality, the error vector can be written:

$$(\epsilon'_i, u_{1i})' = C\nu_i \quad \nu_i \sim N(0_T, I_T) \quad (14)$$

Dropping the  $i$  index:

$$y_T = \mathbf{1}(\lambda y_{T-1} + x_T\beta + c_{T1}\nu_1 + c_{T2}\nu_2 + \dots + c_{T,T-1}\nu_{T-1} + c_{TT}\nu_T > 0) \quad (15)$$

$$y_{T-1} = \mathbf{1}(\lambda y_{T-2} + x_{T-1}\beta + c_{T-1,1}\nu_1 + c_{T-1,2}\nu_2 + \dots + c_{T-1,T-1}\nu_{T-1} > 0) \quad (16)$$

$$\vdots \quad (17)$$

$$y_2 = \mathbf{1}(\lambda y_1 + x_2\beta + c_{22}\nu_2 + c_{21}\nu_1 \quad (18)$$

$$y_{i1} = \mathbf{1}(x_{i1}\xi_1 + \dots + x_{iT}\xi_T + c_{11}\nu_{i1} > 0) \quad (19)$$

This recursive representation establishes the coherency of the model: given a random draw of  $\nu_{i1}, \dots, \nu_{iT}$ , an unambiguous DGP rule can be defined to establish sequentially  $y_{i1} \rightarrow y_{i2} \rightarrow \dots y_{i,T-1} \rightarrow y_{iT}$ .

## 6 Performance of the Novel Estimation Strategy: Summary of Monte Carlo Evidence

As we showed in the previous section, we obtain a coherent non-recursive model with interaction dummies included on both sides, provided we believe the feedback terms have opposite signs on the two sides. Note that it is sufficient to consider only the  $\gamma \geq 0, \delta \leq 0$  case, since the reverse can always be subsumed by redefining both dependent binary variables to their complements  $y'_{it} \equiv (1 - y_{it})$ . The Monte Carlo experiments performed in this study were designed to illustrate the consequences of adopting existing and novel estimation strategies for the problem of this paper. The experiments confirmed that the conditional likelihood approach under sign restrictions described in the previous section, provides reliable, consistent and efficient estimates of the underlying parameters including the two interaction terms. In contrast, the existing traditional approaches (unrestricted MLE ignoring possible incoherency and MLE that incorrectly assumes recursivity of the system) give seriously misleading and inconsistent results. For an extensive presentation of the Monte Carlos along the lines of this Section and detailed analysis and findings, the interested reader is referred to the on-line working paper [Hajivassiliou, 2008]. Here we give a very drastic summary of our main findings:

- The Conditional Truncated MLE proposed in this paper performs very satis-

factorily, being the only consistent estimator for the reverse feedback cases, and only small sacrifices in terms of efficiency in the recursive DGPs when it is not strictly necessary.

- The linear probability estimators, LPOLS and LOP2SLS, perform very badly in all cases with endogenous interaction terms, thus suggesting that the inherent non-linearities of the bivariate probits cannot be safely ignored.
- Conditional Truncated MLE also works well for the overlap region incoherency cases.
- Unrestricted likelihood estimation ignoring the resulting incoherency due to the empty or overlap region(s) (estimator **E-INCO**) is by far the worst performing estimator, dominated even by equation by equation univariate estimators which ignore the other side of the model.<sup>10</sup>

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<sup>10</sup>Some further important features of [Hajivassiliou, 2008] are:

- The procedure for summarizing the MSE results was as follows: A given chart graphs the performance of each estimation algorithm in terms of a given estimation criterion (e.g., RMSE etc.) with the best performing algorithm normalized to 100. The other methods are then given as a fraction of that best. For example, if method A is the best with RMSE=25, and methods B and C have RMSE equal to 75 and 125 respectively, method A will be reported as 100, B as 0.333 (one third as good since 3 times as high RMSE), and C as 0.20 (one fifth as good since 5 times as high RMSE).
- In a set of four-part figures, the *overall* RMSE results are presented with each method’s performance averaged across all estimated parameters. The CMLE estimator dominates all other methods in impressive fashion when the true DGP possesses the opposite-signs restriction  $\gamma \cdot \delta \leq 0$ . It also performs very satisfactorily in case the true model is recursive, achieving almost as good a performance as the ideal recursive estimator for that case. Even in the case of no interaction terms being present in the true DGP ( $\gamma \cdot \delta = 0$ ), the CMLE estimator loses out in terms of RMSE only because of the higher estimation variance in view of not imposing two true restrictions.
- In other sets of four-part figures, the relative RMSE performance for the  $\delta$  interaction parameter, and of parameters  $\beta_{11}$ ,  $\beta_{22}$ , and  $\rho$  respectively. CMLE also impresses in these sets of results in a similar ranking to the previous point.
- Similar figures summarize the *overall* results in terms of *absolute bias* instead of RMSE, as well as absolute *median* bias. The first set establishes that the CMLE estimator heads and shoulders above all the alternatives in terms of bias, and whenever it is less clearly the preferred estimator, this only caused by higher estimation variance.

It may be noted that the dismal performance of the two estimators based on the Linear Probability approximation would have been alleviated had the average partial probability derivatives been calculated instead of the latent variable coefficients. This is because the LP estimators by construction a constant probability derivative with respect to an explanatory variable, irrespective of the observation values. In our view, such calculations would not be especially interesting since in most empirical LDV studies, investigators wish to allow for such probability derivatives to vary over the range of observations.

## 7 Empirical Application: Two-sided Interactions between Financing Constraints and Firm Innovation

We now apply our novel estimation method developed above to analyse the existence and impact of financing constraints as a possibly serious obstacle to innovation by firms. A large strand of the theoretical literature shows how investment is affected by informational asymmetries about the quality of the investment to be financed or relating to the behaviour of entrepreneurs. Such imperfections increase the cost of external finance and therefore, firms may be credit constrained. Due to their specificities inducing large informational asymmetries and poor collateral, innovative firms are more likely to be hampered by financing constraints. Indeed, public authorities have developed a large range of policies such as fiscal incentives and public financing to support innovation in the private sector. However, empirical evidence of the impact of financial constraints on the behaviour of firms are not easy to obtain, essentially because the notional demand of firms for external finance cannot be observed.

In this paper, direct measures of binding constraints are employed, instead of using traditional indirect proxy variables (like firm wealth, accumulated profits, etc) and we use the new econometric approach to be able to estimate for the first time both direct and reverse interactions between innovation and financial constraints. This direct indicator based on firms' own assessments reported in a firm survey (FIT, Financement de l'Innovation Technologique) defines as credit constrained firms facing one of the following difficulties : (i) unavailability of new financing; (ii) searching and waiting for new financing; and (iii) too high cost of new financing. This survey also provides qualitative information about innovative activities of the firm according to the guidelines of the [OECD and Eurostat, 1997]: a firm that introduces or develops a product or process innovation, or is in process of doing so can be characterized as innovative (see the data appendix for detailed information on the sources and on the definitions of variables). When dealing with these indicators of innovation and financing constraints, the empirical analysis needs obviously to account for the problem of *coherency*: the propensity to innovate may be affected by financial constraints, and at the same time, innovative firms are likely to face higher financial constraints due to their informational asymmetries with investors and their lower collateral value. Thus, our new econometric approach enables us to deal with both the direct and reverse effects.

Sections 7.1 and 7.2 provide overview of the theoretical origins of financing constraints and summarize the main findings of the previous empirical investigations. In Section 7.3 we discuss the interest for evaluating the existence of financial constraints with a qualitative self-assessed indicator.

Before applying the ML Estimation approach developed above, we start by following the traditional *recursive* approach to estimate the impact of financial constraints on the probability for a firm to be innovative (Section 7.4). We obtain a very significantly negative effect on innovation due to the presence of financing constraints, *ceteris paribus*. We also show that ignoring the endogeneity of the con-

straint indicator together with the endogenous decision of whether a firm wishes to innovate or not, induces very serious upward biases in the estimated coefficient. A satisfactory resolution to an existing paradox is thus produced: not taking correct account of the endogeneity of financing constraints, leads one to incorrectly conclude that presence of financing constraints and innovation are positively correlated (see [Mohnen and Roller, 2005]) for an example of a study where this “paradox” is encountered). When allowing for endogeneity of “Financing Constraints”: the impact of the “Financing Constraints” indicator has an extremely significant statistically *negative* effect on innovation, with the size of the effect almost tripled once endogeneity is introduced. However, as shown in the first part of this paper, this recursive specification that does not allow for both direct and recursive effects is not fully satisfactory as it may also produced biased results. Indeed, applying the novel econometric method in Section 7.5, we find significant direct and reverse effects : financial constraints reduce significantly the likelihood to be innovative and a firm undertaking actively innovative activities raises significantly the probably of it encountering a binding financing constraint, possibly because potential lenders are particularly wary of granting loans to firms of such type because of the extra riskiness involved.

Finally, we merge our dataset with a previous wave of the innovation survey to study the dynamics of the interaction between innovation and financial constraints in Section 7.6. We find evidence of state dependence for both variables: firms tend to innovate continuously rather than occasionally and past financial difficulties are correlated with the present ones even after conditioning on important firm characteristics. Moreover, it seems that firms with current but also past innovative experiences are more likely to find it difficult to finance their current projects. Section 7.7 discusses a plan for future extensions of this research.

## 7.1 Innovative firms and theoretical origins of financing constraints

Many authors have recognized the likely importance of binding financing constraints can have on firm behaviour. Examples include [Fazzari et al., 1988] who investigate the impact of financing constraints on investment, while more recently [Hennessy and Whited, 2007] attempt to quantify how costly constrained external financing is by the use of simulated moments methodology. The leading causes of encountering financing constraints are similar for tangible investment and R&D projects but the difficulties are more pronounced for firms attempting to implement an innovative process or product ([Hall, 2002]).

First, two problems caused by asymmetric information between a firm and its prospective external financiers are highlighted, implying that the celebrated [Modigliani and Miller, 1958], [Modigliani and Miller, 1963] theorem does not apply in such cases. The first such problem is that of “adverse selection” that increases the cost of external finance. Moreover, as stated by [Stiglitz and Weiss, 1981], informational asymmetries about the quality of the investment may even lead to quantity rationing in preference to allocation of funds through interest rates. Concerning innovative investments, these projects are risky (both from technical and commercial sides) and their expected returns are difficult to evaluate, especially for non specialized investors. Due to the

serious problem of adverse selection they are facing, banks may set high interest rates or even refuse to grant the loan.

The second important informational asymmetry problem is “moral hazard” identified by [Jensen and Meckling, 1976], which is most serious for “start-up” firms, since for such newly established entities the usual dichotomy between owners’ and managers’ interests becomes even more pronounced. Indeed, as small innovative firms are characterized by very poor collateral, high risk but also high expected returns, they are not able to raise funds from traditional investors (in particular banks), while equity financing provided by venture capital is particularly suitable for these start-up firms. The interaction between managers and owners of start-up firms is studied in several theoretical papers that propose incentives such as convertible bonds or stage financing to reduce moral hazard problems caused by the specificities of these new innovative firms (for instance, [Cornelli and Yosha, 2003]).

The next leading cause is due to the fact that innovative firms typically have a higher fraction of intangible assets, implying they are not as able to raise the necessary collateral for low-cost financing. For example, special human expertise and other specific human capital may not be readily marketable outside a specific firm and hence bankers may be unwilling to grant loans based on such assets. Indeed, the capital structure of firms seems to be affected by the proportion of pledgeable assets they own: [Rajan and Zingales, 1995] obtain a positive correlation between the leverage of a firm and the ratio of fixed assets to total assets. Moreover, it has been recently showed that asset tangibility is crucial for identifying financially constrained firms ([Almeida and Campello, 2007]).

## 7.2 Previous empirical investigations

Empirical analysis of credit constraints encountered by firms is complicated by the fact that latent demand for external finance is not observed. Consequently, indirect approaches are mainly adopted to evaluate the impact of financial constraints on innovation.

A near consensus in the literature accepts a significantly positive correlation between firm wealth and investment, and takes this as evidence that firm wealth relaxes financing constraints. It is explained how accumulated wealth and ploughed-back past profits can play an important role in alleviating constraints, since they make internal financing more readily available to the firm.

Following the methodology initiated by [Fazzari et al., 1988], the effect of financial constraints on innovation is evaluated through the impact of cash-flow on R&D investments of sub-groups of firms defined as likely financially constrained or unconstrained. This distinction is based on *a priori* criterion such as dividends distribution, firm age or size. The estimates of the impact of cash-flow on R&D investment are not as conclusive as one might expect. Some authors find a significant positive effect of cash-flow on R&D: [Himmelberg and Petersen, 1994] for small high tech US firms or [Mulkay et al., 2001] for US and French firms. [Harhoff, 1998] find a small significant positive effect of cash flow for German manufacturing firms with a simple profit accelerator investment model but this result is not confirmed by the estimation of

the investment Euler equation. Finally, [Bond et al., 2003] do not obtain significant cash-flow effects on R&D investment for German as well as for UK firms.

It should be self-evident, however, that the traditional approaches of testing for presence of binding financing constraints indirectly by checking for the importance of firm wealth, past profits, etc., can be seriously misleading. For example, it could be that richer firms may *wish* to invest more and hence need more funds simply because they anticipate future profits, not just because they are *able* to borrow more because they face lower constraints. A simple analogy with studying the intertemporal consumption decision is quite illuminating: the Euler equation framework with rational expectations implies that current consumption decisions should be unaffected by current income. The traditional financial constraints empirical analyses are akin to concluding that economic agents face binding liquidity constraints by observing econometrically that current consumption decisions are significantly affected by the level of current income, which of course does not necessarily follow. This line of objection forms the basis of the [Kaplan and Zingales, 1997] critique. The main findings of [Moyen, 2004] are also particularly relevant in this respect: using a synthetic sample methodology, she illustrates that the magnitude and the *sign* of the impact of cash flow on investment change drastically depending on which indirect method is used for inferring the presence of financial constraints on a firm. The pitfalls in using indirect measures of external finance constraints are also highlighted by [Hennessy and Whited, 2007]. [Almeida and Campello, 2007] argue that the identification of the effect of financial constraint can be done thanks to asset tangibility which increases both the ability to raise external financing and the investment of firms facing credit constraints. With a switching regression where constrained firms are endogenously selected, they find that asset tangibility increases investment cash-flow sensitivity for financially constrained firms while no significant effect can be observed for unconstrained firms. Recently, [Gatchev et al., 2010] argue that investment and financial decisions need to be studied simultaneously to properly assess whether firms are constrained from accessing capital market. They conclude that the positive relation between investment and cash flow disappears when accounting for the interdependence of the policy variables.

Instead, in this paper the existence of constraints is not deduced indirectly through the common arguments above, but is directly measured by employing real data on the encountering of binding financing constraints as reported by firms in surveys by Eurostat, carried out since 1990 (Community Innovation Survey, CIS), as well as in a French survey about the financing of innovation (Financement de l'Innovation Technologique, FIT).<sup>11</sup> We argue below that employing such direct measures offers significant benefits for the empirical analysis.

### 7.3 A direct measure of financial constraints

Due to the serious drawbacks of indirect approaches, direct measures of financial difficulties reported by firms can be useful, but very few surveys collect such infor-

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<sup>11</sup>See [Mairesse and Mohnen, 2010] for a review of the use of these surveys.

mation<sup>12</sup>. For instance, [Guiso, 1998] uses a direct qualitative measure given by a survey conducted by Banca d’Italia. In this paper, a firm is characterized as credit constrained “if at the rate of interest prevailing in the loan market, it would like to obtain a larger amount of loans but cannot”. Such a precise definition of credit constrained firms is obtained thanks to the survey used where three questions are asked about access to credit (i) whether at the current market interest rate the firm wish a larger amount of credit, (ii) whether the firm would be willing to obtain more credit, (iii) whether the firm has applied for credit but has been turned down by the financial intermediary. Thanks to this information, the probability to be credit constrained is estimated which leads to the finding that low-tech firms are less likely to be financially constrained than high-tech firms.

In the survey we use (FIT, Financement de l’Innovation Technologique) firms are asked whether some of their innovative projects were delayed, abandoned or non started because of (i) unavailability of new financing, (ii) searching and waiting for new financing, (iii) too high cost of finance. We define as financially constrained firms with hampered innovative projects because of one these three reasons so that our direct indicator of financial constraints takes into account both quantity rationing and higher cost of finance.

In our sample, the quasi-totality of financially constrained firms (88%) suffer from unavailability of new financing. Slowness in the setting up of the financing and too high cost of finance were mentioned respectively by 44% and 23% of constrained firms (see Table 1). The first consequence of these financial obstacles is to prevent the firms from starting their innovative projects. For example, about 58% of firms claiming they were facing too high cost of finance were not able to start the innovative projects. These projects are more rarely only delayed due to financial obstacles: it concerns less than 16% of constrained firms by each obstacle.

Table 1. Financial obstacles and their consequences

	% of Constrained Firms	Consequences for their Innovative Project(s)		
		delayed	abandoned	non started
Unavailability of new financing	87.74	46.27	10.45	46.27
Searching and waiting for new financing	43.23	35.29	12.13	57.72
Too high cost of finance	22.90	28.17	15.49	57.75

Source: FIT survey, French Ministry of Industry

Comparisons across industries show the heterogeneity of the rates of innovative and of financially constrained firms within the manufacturing sector (See Table 2).

<sup>12</sup>We are currently investigating the availability of such direct measures of overall financing constraints from other sources, notably the Banque de France and the French National Institute of Statistical and Economic Studies (INSEE).

Moreover, it seems that financially constrained firms for innovative projects are more likely to belong to innovative industries. Finally, we can notice that the innovation indicator is correlated with the R&D effort in the sector (measured by the ratio of R&D expenditure to sales).

Table 2. Heterogeneity within the manufacturing industry

	Number of firms in the sample  (a)	% of innovative firms within the sector  (a)	% of constrained firms within the sector  (a)	R&D/Sales (%)  (b)
Clothing articles, leather products	100	19.00	10.00	1.78
Publishing, printing, reproduction etc.	101	25.74	12.87	1.21
Pharmaceuticals products, perfumes, etc.	56	53.57	8.93	6.52
Domestic equipment	148	43.92	14.19	2.65
Motor vehicles	75	56.00	16.00	2.48
Building of boats, railway locomotives, etc.	27	62.96	37.04	5.98
Metal products, machinery and equipment	335	48.06	17.31	2.61
Electric and electronic equipment	101	64.36	31.68	9.20
Mining, other non-metallic mineral products	101	37.62	18.81	1.72
Textiles	100	32.00	14.00	1.92
Wood, pulp, paper, etc.	169	27.81	9.47	0.70
Chemicals, rubber, plastic and chemical products	204	43.14	12.25	3.18
Basic metals and fabricated metal products	333	38.14	15.92	1.31
Electric and electronic components	90	60.00	24.44	5.42
Total	1940	41.8	16.0	4.11

(a): in our sample FIT\*CdB (Sources: FIT survey (French Ministry of Industry), CdB (Banque de France))

(b): For the exhaustive population of manufacturing firms in France (Source: Eurostat).

One may argue that direct indicators are subjective and reflect only what firms want to tell: they can claim they encounter financial difficulties to incite public authorities to develop public support to innovation. In order to check whether our indicator fits with economic and financial situation of the firms, we examine the distribution of some ratios reflecting financial health for constrained and unconstrained firms (Table 3). Firm size is not significantly different for constrained and unconstrained firms in our sample. This is not surprising, as smaller firms (with less than 20 employees) are not covered by the survey, while they are the most likely to have difficulties to raise external funds. All other ratios, however, show that constrained firms are characterized by a weaker financial structure and lower economic performance. Financially constrained firms exhibit a higher leverage ratio (for long

term as well as for short term bank loans) that may increase their risk of failure. In particular, the average ratio of short term bank loans that amounts respectively to 89% for constrained firms and to 22% for unconstrained ones is often perceived by financial analysts as a signal for financial difficulties. Moreover, profit margins (measured by the ratio of EBITDA to sales) are more limited for constrained than for unconstrained firms (respectively, 10.8% and 21.6% on average). In addition, constrained firms seem to be less able to generate cash-flow that would compensate a lack of external resources. Finally, the immaterial expenditure rate is higher for constrained firms. It can be due to the fact that firms with large immaterial investments (R&D, patents and marketing) have more difficulties to provide tangible assets to secure bank loans.

To sum up, the direct indicator given by the firms is in line with the balance sheet data: firms without financial constraints exhibit a better profile than constrained firms in terms of financing structure, risk and economic performances.

Table 3. Direct indicator and balance sheet ratios

	Constrained firms				Unconstrained firms			
	Q1	Q2	Q3	Mean	Q1	Q2	Q3	Mean
Number of employees	47	112	290	249.7	47	102	243	227.6
Debt/Equity	8.4	50.7	147.9	132.7	7.9	3.4	9.2	55.0
- Long term bank debt/Equity	0.6	21.9	62.3	43.7	1.7	18.6	51.9	33.4
- Short term bank debt/Equity	0.1	15.4	73.5	89.1	0.0	3.7	34.9	21.6
EBITDA/Sales	6.6	15.4	25.4	10.8	11.8	20.8	30.5	20.3
Cash-flow/Total assets	2.9	7.3	11.1	5.8	5.2	8.5	12.2	8.8
Immaterial Inv/Value added	0.4	1.5	3.9	4.6	0.3	1.1	3.0	3.3

Sources: FIT survey (French Ministry of Industry), CdB (Banque de France)

In the following sections, we turn to our main issue by examining the interaction between financial constraints and innovation.

#### 7.4 Propensity to innovate and determinants of financial constraints: a "traditional" analysis

Main determinants of the propensity for a firm to innovate are known to be its size, its market power and its environment ([Cohen and Levin, 1989]).

The positive correlation between innovation and firm size is largely exposed in the literature (see [Cohen and Klepper, 1996]). Large firms can amortize sunk costs caused by their innovative activities by selling more units of output than smaller

firms. In addition, they are able to diversify the risk incurred by innovation by running simultaneously several investment projects at the same time. And finally, large established firms are less likely to be financially constrained as they are able to generate cash-flow and to raise external funds.

The link between innovation and competition is not well established both from the theoretical and empirical point of view. The Schumpeterian theory argues that market power and innovation are positively correlated whereas Arrow's theory shows that the gains to innovate are larger in an ex-ante competitive market. More recently, [Aghion et al., 2005] try to solve this puzzle and propose an inverted U shape relationship between innovation and competition : in a competitive environment, firms are incited to innovate to gain market power and increase their profits, but when competition becomes hard, the followers can be discouraged to innovate.

Other factors affecting innovative behaviour are driven by the firm environment. The technologic push that results from a lot of various factors such as the state of art, the process of diffusion of knowledge, the connections to academic research centers, etc., leads firms to develop or adopt innovative processes and products. The decision to innovate is also linked to the latent consumer demand perceived by the firm.

We take as our starting point the results obtained by [Savignac, 2006] who studied the impact of financial constraints on the decision to innovate by investigating the impact of financial constraints on innovation without allowing for the probability of a binding finance constraint to depend on whether or not the firm is innovative. The propensity to innovate is explained by the traditional determinants of innovation exposed above (firm size and market power, technology push, latent consumer demand) and we account for financial constraints thanks to our qualitative indicator reflecting the financial difficulties encountered by firms to conduct their innovative projects.

#### **7.4.1 Financial constraints: latent variable versus binary indicator**

First, a simple probit model without taking into account financial constraint is estimated (Model 0 in Table 4.a). Concerning the impact of financial constraints on the propensity to innovate, two specifications can be envisaged : either considering the latent variable that represents the "severity" of financial constraints, or including our binary indicator of financial constraints. See footnote 8 for details.

**Table 4.a**  
**Propensity to Innovate Probit Ignoring Endogeneity**  
**(full sample, nobs=1940)**

Variable	Model 0		Model 1		Model 2	
	Coeff.	Std.	Coeff.	Std.	Coeff.	Std.
Constant	-2.51***	0.21	-2.73***	0.26	-2.61***	0.21
Size	0.32***	0.03	0.23***	0.06	0.33***	0.03
Market share	-0.01	0.06	-0.03	0.06	-0.01	0.06
TP4	1.76***	0.15	1.74***	0.15	1.66***	0.16
TP3	1.25***	0.12	1.22***	0.12	1.19***	0.12
TP2	0.82***	0.12	0.80***	0.12	0.77***	0.12
Financial constraints	-	-	-	-	0.55***	0.09
Collateral amount	-	-	0.08*	0.041	-	-
Banking debt	-	-	-0.001	0.001	-	-
Own financing	-	-	0.001	0.001	-	-
Profit margin	-	-	0.004**	0.002	-	-
Industry dummies	misc		misc		misc	
Log lik	-1080.5		-1073.2		-1060.3	

**Table 4.b**  
**Propensity to Innovate Probit Ignoring Endogeneity**  
**(Potentially innovative firms, nobs=1082)**

Variable	Model 0		Model 1		Model 2	
	Coeff.	Std.	Coeff.	Std.	Coeff.	Std.
Constant	-1.22***	0.34	-1.11**	0.51	-1.07***	0.35
Size	0.27***	0.06	0.27***	0.06	0.27***	0.06
Market share	0.76***	0.30	0.60**	0.29	0.72**	0.30
TP4	1.20***	0.24	1.18***	0.24	1.33***	0.24
TP3	0.81***	0.20	0.76***	0.20	0.88***	0.20
TP2	0.39***	0.19	0.33*	0.19	0.44**	0.19
Financial constraints	-	-	-	-	-0.52***	0.10
Collateral amount	-	-	-0.002	0.002	-	-
Banking debt	-	-	-0.007	0.004	-	-
Own financing	-	-	0.002	0.004	-	-
Profit margin	-	-	0.007***	0.002	-	-
Industry dummies	misc	misc	misc	misc	misc	misc
Log lik	-516.2		-504.8		-501.9	

The first solution leads to the following equations:

$$I_i = \begin{cases} 1 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i^* + \epsilon_i^I > 0 \\ 0 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i^* + \epsilon_i^I \leq 0 \end{cases}$$

with  $F_i^* \equiv x_i^F \beta^F + \epsilon_i^F$

$I_{it}$  is a binary indicator equals to one if the firm  $i$  is engaged into innovative activities, equals to zero otherwise,  $x_i^I$  are the traditional determinants of innovation (firm size, market power, technology push indicators and industry dummies) and  $x_i^F$  are variables reflecting firm ex ante financing structure (banking debt and own financing ratios), profit margins and collateral.

Hence, following the derivations in footnote 8, the reduced form equation for the propensity to innovate is given by:

$$I_i^* \equiv x_i^I \beta^I + \gamma(x_i^F \beta^F + \epsilon_i^F) + \epsilon_i^I$$

Considering a latent variable representing financial difficulties leads to the following probit model:

$$I_i = \begin{cases} 1 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma(x_i^F \beta^F + \epsilon_i^F) + \epsilon_i^I > 0 \\ 0 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma(x_i^F \beta^F + \epsilon_i^F) + \epsilon_i^I \leq 0 \end{cases} \quad (20)$$

The second solution leads to consider the binary indicator as the explanatory variable of the propensity to innovate. It leads to the following equation:

$$I_i = \begin{cases} 1 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i + \epsilon_i^I > 0 \\ 0 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i + \epsilon_i^I \leq 0 \end{cases} \quad (21)$$

where  $F_i$  is the binary indicator reflecting the fact that the firm faces financial constraints.

With the specification (20) in terms of latent variable, we obtain a significant positive impact of profit margins that may reflects the effect of binding financing constraints. However, as presented before, interpreting this positive coefficient as revealing the existence of financial constraints remains ambiguous. Moreover, there are also other economic reasons to render this model unsatisfactory where the investment outcomes depend on the continuous latent variable. The economic agents in our setting base their decisions on relevant observable outcomes, since the latent propensities of the other side cannot be observed by them. Firms' strategy to choose to innovate will be affected by whether or not the bank they approached does or does not grant them the requested financing. It is irrelevant to the firm's decision (and also unobservable) the *extent* to which the bank came close to granting the loan request — the latter being measured by the latent bank propensity of granting the loan.

Therefore, we prefer to model the investment outcomes as depending on the discrete *outcome* from the other side (model 21), instead of on the continuous latent

variable of the other side (model 20). Nevertheless, with a simple probit, a surprising significant positive effect of the financial constraint is obtained (Column 3 of Table 4.a). This positive effect is explained by two sources of bias that we tackle below: a selection bias due to firms not wishing to innovate and a problem of simultaneity between investment and financing decisions.<sup>13</sup>

#### 7.4.2 The Importance of Sample Selection and Simultaneity

**Sample selection** The sample selection problem is due to firms that do not wish to innovate, and consequently do not face binding financing constraints. A simple example is quite enlightening to understand the nature of this bias and the way to tackle it. Suppose one half of the firms do not wish to innovate, and hence do not face binding financing constraints by banks. The other half of firms who wish to innovate, approach the banks for a loan. For simplicity, let us say that half of them face a binding constraint and are denied their request for loan by the bank, while the other half are granted their request. Consequently, three quarters of firms end up *not* innovating (1/2 who did not wish to innovate and did not face finance constraints (group A), plus 1/4 who wished to innovate but were refused the loan (group B)), while only 1/4 end up innovating (wished to and were granted their loan request (group C)). Group A of size 1/2 exhibits positive correlation between constraints and innovation (did not innovate, did not face constraints), while groups B of size 1/4 and C of size 1/4 exhibit negative correlation (B: did not innovate because of high financing constraints, C: innovated because of low constraints). If we select only the potentially innovating firms (B+C) the correlation is very negative. If we average also group A, the overall correlation will be significantly higher, namely 0 in this example.

Therefore, we isolate the firms that wished to innovate by excluding non innovative firms without any obstacles to innovation (financial constraints as well as other factors collected by the survey which may limit innovation such as: market risk, unavailability of suitable personnel, not sufficient knowledge of available sources of finance). The obtained subsample is composed of the potentially innovative firms, namely the groups B and C in the previous example. As expected, the estimated correlation between innovation and financing constraints becomes significantly lower and negative by restricting the econometric sample to potentially innovative firms (Table 4.b).

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<sup>13</sup>We conduct some robustness checks concerning the definition of the explanatory variables. Here, size is measured by the log of the number of employees to account for non-linear effects. We also test total assets as a measure of firm size both in terms of log and in level with a squared term. All these definitions lead to similar results. To distinguish between financial and *economic* distress it may be useful to add other liquidity measures such as available cash stock. However, as this variable is strongly correlated with profit margins it is redundant and does not provide significant results when it is introduced together with profit margins. Belonging to a holding group is likely to help innovation or to lead to less financial distress but we were not able to find such significant effects with our data, probably because they do not allow to account for *the size of the holding group* nor to identify the financial connections within the holding group. Finally, the likely endogeneity of the profitability and asset tangibility measures needs to be addressed.

**Simultaneity and the Econometric Problem of “Coherency” in LDV Models** Obviously, the decision to innovate and the difficulties encountered to finance the project occur simultaneously which leads the financial constraint variable to be endogenous in the innovation equation. A simple way to tackle this problem is to define a simultaneous probit model where the probability that a firm decides to innovate and the probability that a firm faces a binding financing constraint are simultaneously estimated. Such a system can be formulated as follows:

$$I_i = \begin{cases} 1 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i + \epsilon_i^I > 0 \\ 0 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i + \epsilon_i^I \leq 0 \end{cases} \quad (22)$$

$$F_i = \begin{cases} 1 & \text{if } F_i^* \equiv x_i^F \beta^F + \delta I_i + \epsilon_i^F > 0 \\ 0 & \text{if } F_i^* \equiv x_i^F \beta^F + \delta I_i + \epsilon_i^F \leq 0 \end{cases} \quad (23)$$

which was presented in Section 2.2 as equations (3)-(4). However, as presented in the theoretical part of this paper, such a model with qualitative endogenous variables and reverse effects is incoherent and cannot be estimated ([Gourieroux et al., 1980], [Lewbel, 2007]). The formal condition for the econometric “coherency” of this model obtained in the existing literature is that  $\gamma \cdot \delta = 0$ , which of course corresponds to no reverse interaction terms existing between the innovation and financing constraint side: if  $\gamma \neq 0$  and binding financing constraints are allowed to affect the probability that a firm innovates in the innovation equation, the innovation indicator is not allowed to be included in the financing constraint equations and vice versa (if  $\delta \neq 0$ ,  $\gamma$  must be 0). Of course, this solution is not satisfactory because there are no economic reasons to choose one or the other recursive specification as both financial constraints may hamper innovation by firms and innovative firms may encounter reinforced financial constraints.

Before applying in the next section our novel econometric approach that allows us to estimate both direct and reverse effects, we present here the results obtained with the usual recursive specification. We consider the case where  $\delta = 0$  and include the financial constraint indicator in the innovation equation. The dataset includes some information about other distress factors, such as: market risk, unavailability of suitable personnel, too high costs, excessive costs for abandoning innovation, and not sufficient knowledge of available sources of finance. From the econometric point of view, these other obstacles raise the same problem of endogeneity as the financial constraint variable and a multivariate simultaneous model with one equation explaining the probability of encountering each obstacle would be necessary to properly estimate the impact of these other obstacles on innovation. Due to the size of such a model and the various explanatory variables needed that are not available in our dataset, a simpler way to proceed consists in building a composite indicator of the various obstacles. But we leave this question for the future and focus here on the impact of financial constraints on innovation.

In sharp contrast with the univariate model, when endogeneity is accounted for, the estimated coefficient of the financial constraints becomes negative, as expected on a priori grounds, while all other estimates remain largely unchanged (second column

of Tables 5.a and 5.b). Furthermore, a strong correlation between the errors of both equations is found when both equations are simultaneously estimated. Therefore, the results obtained here show without ambiguity that innovation is hampered by financial constraints, while previous papers based on the correlation between investment and wealth were subject to interpretation problems. As stated before, however, the estimated model remains not entirely satisfactory because it does not allow for reverse effects, although the propensity to innovate may be affected by financial constraints, and at the same time, innovative firms are likely to face higher financial constraints due to informational asymmetries with investors and their lower collateral value. Our analysis of the next Section enables us to overcome these critical shortcomings.

**Table 5.a**  
**Innovation and Financing Constraints Joint Probit**  
**Without Reverse Interaction Effects (full sample, nobs=1940)**

	Single Equations		Bivariate Probits	
	Coeff.	Std.	Coeff.	Std.
<b>Innovation equation</b>				
Constant	-2.61***	0.21	-2.121***	0.278
Size	0.33***	0.03	0.305	0.034
Market share	-0.01	0.06	-0.001***	0.055
TP4	1.66***	0.16	1.504***	0.170
TP3	1.19***	0.12	1.068***	0.134
TP2	0.77***	0.12	0.690***	0.121
Financial Constraints	0.55***	0.09	-0.550**	0.268
11 Industry dummies	misc		misc	
<b>Financial Constraint Equation</b>				
Constant	-0.868***	0.243	-0.816***	0.237
Size	-0.054	0.067	-0.002	0.070
Collateral amount	0.067	0.047	0.030	0.048
Banking debt ratio	0.010***	0.002	0.010***	0.002
Own financing ratio	-0.003**	0.001	-0.003***	0.001
Profit margin	-0.007***	0.001	-0.008***	0.002
11 industry dummies	misc		misc	
<i>corr</i> <sub>12</sub>	-	-	0.572***	0.161
Log lik Innovation	-1060			
Log lik Fin Constraint	-803			
Log lik Bivariate			-1088	

**Table 5.b**  
**Innovation and Financing Constraints Joint Probit**  
**Without Reverse Interaction Effects**  
**(Potentially innovative firms, nobs=1082)**

	Single Equations		Bivariate Probits	
	Coeff.	Std.	Coeff.	Std.
<b>Innovation Equation</b>				
Constant	-1.074***	0.347	-0.496	0.365
Size	0.268***	0.572	0.224***	0.057
Market share	0.715**	0.298	0.628**	0.243
TP4	1.331***	0.243	1.195***	0.229
TP3	0.879***	0.198	0.770***	0.182
TP2	0.439**	0.192	0.371**	0.172
Financial Constraints	-0.524***	0.098	-1.380***	0.247
11 Industry dummies	misc		misc	
<b>Financial Constraint Equation</b>				
Constant	0.588*	0.324	0.581*	0.329
Size	-0.085	0.083	-0.036	0.084
Collateral amount	0.001	0.059	-0.034	0.060
Banking debt ratio	0.005	0.003	0.006*	0.003
Own financing ratio	-0.012***	0.003	-0.012***	0.003
Profit margin	-0.009***	0.002	-0.009	0.002
11 industry dummies	misc		misc	
<i>corr</i> <sub>12</sub>	-	-	0.572***	0.161
Log lik Innovation	-501			
Log lik Fin Constraint	-591			
Log lik Bivariate				-1088

## 7.5 Empirical Results Establishing Reverse Interaction Effects

Novel econometric developments in the first part of this paper prove that this condition for coherency is not necessary, however. More precisely, the new results establish that our bivariate binary probit model can be estimated efficiently through the Truncated Maximum Likelihood method, conditional on the prior sign restriction that  $\gamma$  and  $\delta$  must not have the same sign. On a priori economic reasoning grounds, we expect this sign restriction to be satisfied in our case, since we believe that  $\gamma \geq 0$  (a binding finance constraint lowers the likelihood that a firm will afford to innovate ceteris paribus), while  $\delta \leq 0$  (innovative firms are more likely to face binding financing constraints due to informational asymmetries ceteris paribus).

Using this econometric machinery that allows us to estimate joint binary probit equations with interaction terms on both sides, we present in Tables 6.a and 6.b the application of those methods to the key issue of Being Innovative vs. Binding Financing Constraints interactions. Our final estimates in the second columns of each

table confirm our prior expectations, as well as the Monte Carlo findings summarized in the Technical Appendix: a firm undertaking actively innovative activities raises significantly the probability of it encountering a binding financing constraint, possibly because potential lenders are particularly wary of granting loans to firms of such type because of the extra riskiness involved. The effect is stronger for the full sample of firms, presumably because the greater homogeneity of the potentially innovative firms subsample dampens the impact of this interaction. The inclusion of interaction terms in both sides results in a lowering of the significance of the estimated correlation coefficients in the unobservables of the two sides, which is reassuring. Our findings should act as a strong warning to researchers in this field who employ traditional methods that either ignore or incorporate inappropriately the model coherency issue: the resulting estimation biases from such practices appear very serious indeed.<sup>14</sup>

**Table 6.a**  
**Innovation and Financing Constraints Joint Probit**  
*With Reverse Interaction Effects (full sample, nobs=1940)*

	Single Equations		Bivariate Probits	
	Coeff.	Std.	Coeff.	Std.
<b>Innovation equation</b>				
Constant	-2.731***	0.225	-7.235***	0.118
Size	0.304***	0.034	0.183***	0.020
Market share	0.025	0.063	0.020	0.045
TP4	1.646***	0.165	1.822***	0.183
TP3	1.086***	0.132	1.0110***	0.199
TP2	0.684***	0.128	0.437***	0.176
Financial Constraints	0.127	0.105	-0.324**	0.255
11 Industry dummies	misc		misc	
<b>Financial Constraint Equation</b>				
Constant	-0.868***	0.243	-1.221***	0.241
Firm Innovates	-	-	0.647***	0.032
Size	-0.054	0.067	-0.016	0.073
Collateral amount	0.067	0.047	0.030	0.050
Banking debt ratio	0.010***	0.002	0.015***	0.002
Own financing ratio	-0.003**	0.001	-0.001***	0.001
Profit margin	-0.007***	0.002	-0.002***	0.002
11 industry dummies	misc		misc	
<hr/>				
<i>corr</i> <sub>12</sub>	-	-	-0.132***	0.013
Log lik Innovation	-965.4			
Log lik Fin Constraint	-803.7			
Log lik Bivariate			-1712	

<sup>14</sup>Furthermore, we obtained but do not report results that show that attempting to apply Linear Probability methods to the Innovation-Financing Constraint interaction leads to very significant biases also. This finding is in line with the Monte-Carlo results of Section 6

**Table 6.b**  
**Innovation and Financing Constraints Joint Probit**  
*With Reverse Interaction Effects*  
**(Potentially innovative firms, nobs=1082)**

	Single Equations		Bivariate Probits	
	Coeff.	Std.	Coeff.	Std.
<b>Innovation Equation</b>				
Constant	-0.879**	0.356	-0.292	0.388
Size	0.283***	0.058	0.232***	0.060
Market share	0.698**	0.295	0.643***	0.240
TP4	1.343***	0.249	1.210***	0.238
TP3	0.871***	0.203	0.766***	0.188
TP2	0.431**	0.197	0.363**	0.179
Financial Constraints	-0.415***	0.109	-1.290***	0.269
11 Industry dummies	misc		misc	
<b>Financial Constraint Equation</b>				
Constant	0.458	0.417	0.467	0.410
Firm Innovates	-	-	0.324***	0.133
Size	-0.150*	0.081	-0.099	0.084
Collateral amount	0.069	0.058	0.030	0.059
Banking debt ratio	0.007	0.004	0.004*	0.004
Own financing ratio	-0.007*	0.004	-0.003**	0.003
Profit margin	-0.012***	0.002	-0.077***	0.002
11 industry dummies	misc		misc	
<i>corr</i> <sub>12</sub>	-	-	0.254***	0.150
Log lik Innovation	-488.8			
Log lik Fin Constraint	-599.2			
Log lik Bivariate			-1067.9	

## 7.6 State Dependence in Financing and Innovation Experiences of Firms

We now explain how the nature of the available datasets can be exploited to study whether, *ceteris paribus*, past financial distress or innovation failures can affect a firm's current experiences in these two dimensions.

Though the surveys about innovation we use are not truly longitudinal "panel" sets, the information we use was collected in four biennial waves. Hence, we know whether a particular firm  $i$  has reported binding financing constraints in the past. Similarly, we can also tell whether a firm has failed in the past in its efforts to be innovative. Consequently, though our dataset is not a true panel, we can extend our econometric equations modelling the probabilities of being innovative and of encountering binding financing constraints at the end of the sample period to condition also on past experiences in these two dimensions.

The sample design of the survey does not allow to observe every firm in the four

waves, therefore we restrict our “longitudinal” panel to two waves in order to limit the reduction of the sample size and the sample bias when merging the waves (see the transition tables in the Data Appendix).

We estimate the following model that accounts for the dynamic effects:

$$I_{it} = \begin{cases} 1 & \text{if } I_{it}^* \equiv \alpha^I I_{it-1} + x_{it}^I \beta^I + \gamma_0 F_{it} + \gamma_1 F_{it-1} + \epsilon_{it}^I > 0 \\ 0 & \text{if } I_{it}^* \equiv \alpha^I I_{it-1} + x_{it}^I \beta^I + \gamma_0 F_{it} + \gamma_1 F_{it-1} + \epsilon_{it}^I \leq 0 \end{cases} \quad (24)$$

$$F_{it} = \begin{cases} 1 & \text{if } F_{it}^* \equiv \alpha^F F_{it-1} + x_{it}^F \beta^F + \delta_0 I_{it} + \delta_1 I_{it-1} + \epsilon_{it}^F > 0 \\ 0 & \text{if } F_{it}^* \equiv \alpha^F F_{it-1} + x_{it}^F \beta^F + \delta_0 I_{it} + \delta_1 I_{it-1} + \epsilon_{it}^F \leq 0 \end{cases} \quad (25)$$

In summary, our findings in Tables 7.a and 7.b confirm the very strong importance of such dynamic terms and establish very significant positive state dependency in our models. First, our results show that firms tend to innovate persistently rather than occasionally.

Second, past financial difficulties are positively correlated with current binding financial constraints. As we take into account the experience of a firm concerning innovation, the state dependence of financial constraints seems particularly interesting. Indeed, firms currently implementing innovative projects as well as firms with innovative experience in the past are more likely to find it difficult to finance their current projects.<sup>15</sup>

Third, the probability for a firm to be currently conducting an innovative project is negatively impacted by the current financing difficulties as founded in the static regressions but also positively correlated with financing constraints encountered in the past. One possible explanation for this positive correlation could be that financial difficulties mainly impact the beginning of the projects so that innovative projects that were initially hampered by financial difficulties are more likely to be continued when they become more mature. However, additional information on the stage of development of the innovative projects and on their duration would be necessary to further investigate this point, in particular we are not able to identify whether the firms were continuing in 1997-1999 projects that were already conducted in 1994-1996.

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<sup>15</sup> An important issue discussed frequently in the econometrics literature is the possibility that state dependence may not be an important factor *per se*, but it might appear statistically significant if persistent heterogeneity among individual economic agents is ignored. As [Heckman, 1981b] shows, the two can be identified when a panel data set with more than two waves per individual is available. Since our dynamic sample consists only of two waves, we need to acknowledge the possibility that the strong state dependence we report here may be compounded by unobserved persistent heterogeneity that is not accounted for explicitly.

**Table 7.a**  
**Innovation and Financing Constraints Joint Probit**  
*With Reverse Interaction Effects and Dynamics*

(full sample, nobs=1512)				
	Single Equations		Bivariate Probits	
	Coeff.	Std.	Coeff.	Std.
<b>Innovation equation</b>				
Constant	-2.747***	0.225	-2.441***	0.323
Innov <sub>t-1</sub>	0.888***	0.083	0.829***	0.094
Size	0.262***	0.037	0.256***	0.037
Market share	0.030	0.066	0.027	0.071
TP4	1.550***	0.188	1.461***	0.201
TP3	1.009***	0.147	0.932***	0.156
TP2	0.674***	0.143	0.621***	0.143
Financial Constraints	0.450***	0.105	-0.447	0.396
Financial Constraints <sub>t-1</sub>	0.124	0.097	0.300	0.123
11 Industry dummies	misc		misc	
<b>Financial Constraint Equation</b>				
Constant	-0.814***	0.304	-0.885***	0.311
Firm Innovates <sub>t</sub>	-	-	0.647***	0.032
Financial constraints <sub>t-1</sub>	0.645***	0.091	0.635***	0.093
Size	0.033	0.037	0.035	0.039
Collateral amount	0.002	0.002	0.003	0.002
Banking debt ratio	0.004	0.003	0.005	0.003
Own financing ratio	-0.009***	0.003	-0.008***	0.002
Profit margin	-0.006***	0.002	-0.007***	0.002
11 industry dummies	misc		misc	
<hr/>				
<i>corr</i> <sub>12</sub>	-	-	0.500**	0.210
Log lik Innovation	-742.8			
Log lik Fin Constraint	-596.8			
Log lik Bivariate			-1337.5	

**Table 7.b**  
**Innovation and Financing Constraints Joint Probit**  
*With Reverse Interaction Effects and Dynamics*  
**(Potentially innovative firms, nobs=782)**

	Single Equations		Bivariate Probits	
	Coeff.	Std.	Coeff.	Std.
<b>Innovation Equation</b>				
Constant	-1.231***	0.416	-0.595	0.453
Innov <sub>t-1</sub>	0.735***	0.121	0.645***	0.121
Size	0.250***	0.064	0.201***	0.066
Market share	0.541*	0.292	0.474*	0.272
TP4	1.393***	0.305	1.214***	0.307
TP3	0.664***	0.236	0.548**	0.228
TP2	0.340	0.230	0.261	0.218
Financial Constraints	-0.658***	0.121	-1.676***	0.285
Financial Constraints <sub>t-1</sub>	0.184	0.135	0.452***	0.147
11 Industry dummies	misc		misc	
<b>Financial Constraint Equation</b>				
Constant	0.025	0.372	-0.153	0.380
Firm Innovates	-	-	0.324***	0.133
Firm Innovates <sub>t-1</sub>	-	-	0.122*	0.053
Financial constraints <sub>t-1</sub>	0.670***	0.109	0.674***	0.112
Size	-0.070	0.044	-0.064	0.046
Collateral amount	0.003	0.002	0.003	0.002
Banking debt ratio	0.005	0.004	0.006*	0.004
Own financing ratio	-0.009***	0.003	-0.008**	0.003
Profit margin	-0.009***	0.003	-0.009***	0.002
11 industry dummies	misc		misc	
<hr/>				
<i>corr</i> <sub>12</sub>	-	-	0,675***	0,192
Log lik Innovation	-346.6			
Log lik Fin Constraint	-443.9			
Log lik Bivariate			-786.7	

## 7.7 Avenues for Future Research

In this section we outline our plans for extending the empirical analysis about the financing of innovative enterprises in several directions.

First, there might exist useful additional information in the ranking of the different types of financing constraints over and above the simple binary indicator used here. For example, an ordered 3-value financing indicator might have been used instead, with the second equation being a 3-way ordered probit instead of a binary one.

Second, we could consider splitting the sample of innovative firms into ones who are *real innovators* from the ones who are merely *adopters* of innovative technologies and/or processes. This information will be available in the forthcoming CIS4

survey. On a priori grounds, it appears that financiers are likely to treat these two sub-types of innovative firms as quite distinct, with concomitant differences in funding riskiness. Unfortunately, the sub-category of “potentially innovative” firms who have attempted to introduce innovations but *failed* may need to be dropped altogether and the consequences of doing so to be investigated, since it appears impossible to identify the *reasons* for such failures, in particular whether it was due to binding financial constraints or due to the innovation idea being inherently bad.

A related hypothesis we plan to analyze, which is studied preliminarily in [Corres et al., 2006], is that of *hierarchical financing* in terms of effective cost: first proceed with internal financing, and only continue with external one if the granted interest rate is low enough. Such an analysis would be useful to understand why one generally observed *lower* interest rates on bank debt for innovative firms than for non innovative ones (Savignac, 2006). The first alternative explanation explored is that the structure of types of debt are different, which would explain partially the differences in average interest rates observed. The second alternative is that innovating firms are in general of bigger size and established for longer so that they can be granted loans at lower interest rates. The third alternative explanation, which our preferred one, goes as follows: the findings are driven by a very serious selection bias, since observing a zero interest rate could mean one of three things: (a) the firm did not want any new loan; (b) the firm requested a loan but did not take one because the interest terms of the loan were too expensive; or (c) the firm requested a loan but the bank did not grant one. Along similar lines, it appears worthwhile to study explicitly the alternative mode of financing of issuing new shares for publicly quoted firms.

A further refinement we consider elsewhere is to build more detailed sequential models whereby a firm chooses whether or not to request a loan and given it decides yes, whether or not the bank grants the loan. In addition, it seems interesting to model also the final possibility that a firm may apply for a loan, the bank might grant it, *but the firm might reject the offer as too costly*. Assuming all the necessary data exists for building such an econometric framework, one could learn quite a lot about the mechanisms of these markets.

Finally, it should be noted that our econometric approach developed in this paper can be applied to the related problem of investment decisions by firms being directly affected by binding financing constraints. An important shortcoming of our data is that firms were asked to respond as to whether their *plans to innovate* were thwarted by the presence of financing constraints, and not whether their *overall investment* plans were thus seriously hindered. We are currently investigating the availability of such direct measures of overall financing constraints from other sources, notably the Banque de France and the French National Institute of Statistical and Economic Studies (INSEE). Such direct measures will allow us to answer whether innovative firms encounter more significant financing constraints pertaining to all their plans, not just for funding innovation.<sup>16</sup>

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<sup>16</sup>[Savignac, 2006] finds that innovative firms appear to face significantly reduced probabilities of bank loans specifically because of their innovative activities. In addition, she finds that the bank loans for the innovative firms that were successful in securing them are in fact at a significantly lower

## 8 Conclusions

The paper discussed the major identification issue of *coherency conditions* in LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables. The econometric framework of LDV models with simultaneity was presented and the identification issue of *coherency* in such LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables was analyzed.

Conditions for coherency as presented in the existing literature were reviewed and shown to be rather esoteric. Two novel methods for establishing coherency conditions were presented, one based on a graphical characterization, the second through hypothetical Monte-Carlo DGP. The novel approaches have intuitive interpretations and are easy to implement and generalize. The constructive consequence of the new approaches is that they indicate how to achieve coherency in models traditionally classified as incoherent through the use of prior sign restrictions on model parameters. This allowed us to develop estimation strategies based on Conditional MLE for simultaneous LDV models without imposing recursivity. Thus one can obtain for the first time estimates of direct as well as reverse interaction effects in simultaneous LDV models, unlike in the existing literature where recursivity had to be assumed. Econometric applications were used to illustrate the methods in practice and extensions are given to simultaneous ordered probit models with multiple regions.

The proposed Conditional MLE methodology was evaluated through an extensive set of Monte-Carlo experiments. The experiments allowed us also to study the consequences of employing estimators that make overly restrictive coherency assumptions about the DGP. The findings confirmed very substantive improvements in terms of estimation Mean-Squared-Error by employing the CMLE developed in this paper. They also showed that estimators based on the Linear Probability approximation perform poorly in this context.

Our CMLE approach allows for the first time to obtain estimates of the reverse as well as direct interaction terms in LDV models with simultaneity

In the empirical application of this paper, we proceeded to analyze the existence and impact of financing constraints as a possibly serious obstacle to innovation by firms. Direct measures of financing constraints were employed using survey data collected by the Banque de France and Eurostat, which helped us overcome the problems with the traditional approach in the past literature of trying to deduce the existence and impact of financing constraints through the significance of firm wealth variables.

We used as the main econometric framework for our empirical analyses the simultaneous bivariate probit with mutual endogeneity discussed above, and used our novel methods for establishing coherency conditions that allowed us for the first time to estimate models hitherto classified as incoherent through the use of prior sign restrictions on model parameters. We were thus able to quantify the interaction between financing constraints and a firm's decision and ability to innovate without forcing interest rate on the average.

the econometric models to be recursive. Hence, we obtained direct as well as reverse interaction effects, leading us to conclude that binding financing constraints discourage innovation and at the same time innovative firms are more likely to face binding financing constraints. Finally, we investigated the importance of state-dependence in dynamic versions of our models and concluded that such issues are critical if direct and reverse interactions between innovation and financing constraints are to be quantified reliably.

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## 9 Appendix 1: Generating standard Normal variates truncated to lie outside $[\underline{\lambda}, \bar{\lambda}]$

We present here a method for generating truncated normal variates to ensure the coherency of the non-recursive model under prior sign restrictions:

Let  $z \sim N(0, 1)$  and define  $\tau \sim z | \{z \notin [\underline{\lambda}, \bar{\lambda}]\}$ . Then  $cdf(\tau) : F(\tau) = \Phi(z)$  and

$$cdf(\tau) : F(\tau) = \begin{cases} \frac{\Phi(z)}{1-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})} & \text{if } z < \underline{\lambda}, \\ \frac{\Phi(\underline{\lambda})}{1-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})} & \text{if } \underline{\lambda} < z \leq \bar{\lambda}, \\ \frac{\Phi(z)-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})}{1-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})} & \text{if } z > \bar{\lambda}. \end{cases}$$

The procedure is exact for a univariate  $z$  truncated on  $\{z \notin [\underline{\lambda}, \bar{\lambda}]\}$ , but it will not work for higher dimensions. For DGPs with higher dimensions, accept-reject methods are preferable, though others exist (e.g., Gibbs resampling — see [Hajivassiliou and McFadden, 1998] for an explanation).

## 10 Appendix 2: Design of Monte-Carlo Experiments

The experiments were designed to illustrate the importance of coherency on the following *nine* estimation approaches:

(a) likelihood estimation that incorrectly forces the old coherency condition to hold, i.e., assuming recursivity when in fact both feedback terms are present (estimators E-TRWN=assuming  $\delta = 0$  and E-TRNW=assuming  $\gamma = 0$ );

(b) unrestricted likelihood estimation, which ignores the resulting incoherency due to the empty or overlap region(s) (estimator E-INCO);

(c) restricted likelihood estimation conditioning on the data lying outside the empty region(s) of incoherency (estimators E-SQPM=assuming  $(\gamma \geq 0, \delta \leq 0)$  and E-SQMP=assuming  $(\gamma \leq 0, \delta \geq 0)$ );

(d) restricted likelihood estimation conditioning on the data lying outside the overlap region(s) of incoherency (estimators E-SQPP=assuming  $(\gamma \geq 0, \delta \geq 0)$  and E-SQMM=assuming  $(\gamma \leq 0, \delta \leq 0)$ ).

(e) LPOLS: (linear probability) ordinary least squares estimation of each binary probit equation ignoring the possible endogeneity of the interaction terms; and LP2SLS: applying two-stage least squares recognizing that the two interaction terms on the RHS of each probit equation can be endogenous.

We generate six “true” models:

- DGP-TRWN ( $\delta = 0$ )
- DGP-TRNW ( $\gamma = 0$ )
- DGP-SQPM ( $\gamma \geq 0, \delta \leq 0$ )
- DGP-SQMP ( $\gamma \leq 0, \delta \geq 0$ )
- DGP-SQPP ( $\gamma \geq 0, \delta \geq 0$ ) and
- DGP-SQMM ( $\gamma \leq 0, \delta \leq 0$ ),

and in each case, calculate the nine estimators E-TRWN, E-TRNW, E-INCO, E-SQPM, E-SQMP, E-SQPP, E-SQMM, LPOLS, and LP2SLS.

The generating equations are:

$$y_{star1} = x1[nobs, kx1] * beta1 + gamma * y2 + eps1, \quad y1 = 1(y_{star1} > 0)$$

$$y_{star2} = x2[nobs, kx2] * beta2 + delta * y1 + eps2, \quad y2 = 1(y_{star2} > 0)$$

### 10.1 $\gamma$ unrestricted, $\delta = 0$

$$y_{star1} = x1[nobs, kx1] * beta1 + gamma * y2 + eps1, \quad y1 = 1(y_{star1} > 0)$$

$$y_{star2} = x2[nobs, kx2] * beta2 + eps2, \quad y2 = 1(y_{star2} > 0)$$

Given the recursivity of the  $\gamma \cdot \delta = 0$  restriction in this case,  $y_{star2}$  is generated first, which gives  $y2$ . This is then plugged into the RHS of the  $y_{star1}$  equation thus allowing  $y_{star1}$  and  $y1$  to be obtained.

**10.2**  $\gamma \geq 0, \delta \leq 0$

$$0 \leq eps1 + x1 * b1 \leq gamma, -delta \leq eps2 + x2 * beta2 \leq 0 \quad (26)$$

Accept-reject methods are used to generate the data so that these restrictions are satisfied.

Analogous Accept/Reject DGP for the  $\gamma \geq 0, \delta \geq 0$  case. Also see appendix 1 for an exact algorithm for generating draws from truncated normal distributions restricted to lie on region (26).

**10.3**  $\gamma \leq 0, \delta \geq 0$

$$-gamma \leq eps1 + x1 * b1 \leq 0, 0 \leq eps2 + x2 * beta2 \leq -delta$$

Accept-reject methods are used to generate the data so that these restrictions are satisfied.

Analogous Accept/Reject DGP for the  $\gamma \leq 0, \delta \leq 0$  case.

We performed 24 Monte-Carlo experiments, indexed by MCxyz as follows:

	$\delta$	$\gamma$		
$x = 1$	0	0		
$x = 2$	0.8	0		
$x = 3$	0.8	1		
$x = 4$	0.8	-1		

	$\rho_{\epsilon_1, \epsilon_2}$
$y = 1$	0.3
$y = 2$	-0.3

	$x_{11}$	$x_{12}$	$x_{13}$	$x_{21}$	$x_{22}$	$x_{23}$
$z = 1$	<i>const</i>	$\chi^2(1)$	<i>Bernoulli(0.7)</i>	<i>const</i>	$x_{12}$	<i>DoubleExponentialSS</i>
$z = 2$	<i>const</i>	$\chi^2(1)$	<i>Bernoulli(0.9)</i>	<i>const</i>	$x_{12}$	<i>DoubleExponentialSS</i>
$z = 3$	<i>const</i>	$\chi^2(1)$	<i>Bernoulli(0.7)</i>	<i>const</i>	$x_{12}$	<i>DoubleExponentialLS</i>
$z = 4$	<i>const</i>	$\chi^2(1)$	<i>Bernoulli(0.9)</i>	<i>const</i>	$x_{12}$	<i>DoubleExponentialLS</i>

where *DoubleExponential* stands for a Double Exponential distribution with mean 0 with asymmetric two sides, *SS* for “small skewness” and *LS* with “large skewness.”

All trials used 2000 observations and 200 Monte-Carlo trials were averaged in each case. In all experiments, the true beta parameters were set at:  $\beta_1 = (0.8, -0.5, -0.3)'$  and  $\beta_2 = (-0.3, 0.7, -0.4)'$ . In the next Subsection, we give the complete listing of the regime probabilities in all 32 experiments we carried out.

The full tables presenting the detailed Monte-Carlo results in terms of various estimation criteria (root-mean-squared error, absolute bias, absolute median bias, variance, interquartile range, and nine-decile range) can be obtained from the author upon requests.

## 10.4 Regime Probabilities in Monte-Carlo Experiments

Table 8

	$Y_2 = 1$		$Y_2 = 0$					
$Y_1 = 1$	$p_{11}$	$p_{10}$	$p_{01}$	$p_{00}$	$p_{0\cdot}$	$p_{1\cdot}$	$p_{\cdot 1}$	$p_{\cdot 0}$
$Y_1 = 1$	$p_{01}$	$p_{00}$	$p_{0\cdot}$	$p_{1\cdot}$	$p_{\cdot 1}$	$p_{\cdot 0}$		
	$p_{11}$	$p_{10}$	$p_{01}$	$p_{00}$	$p_{0\cdot}$	$p_{1\cdot}$	$p_{\cdot 1}$	$p_{\cdot 0}$
<b>mc111</b>	0.2812	0.2716	0.1759	0.2711	0.4470	0.5529	0.4572	0.5427
<b>mc112</b>	0.2736	0.2589	0.1845	0.2829	0.4674	0.5325	0.4581	0.5418
<b>mc113</b>	0.2696	0.2840	0.1748	0.2714	0.4463	0.5536	0.4445	0.5554
<b>mc114</b>	0.2598	0.2728	0.1844	0.2829	0.4673	0.5326	0.4442	0.5557
<b>mc121</b>	0.2262	0.3273	0.2316	0.2147	0.4464	0.5535	0.4579	0.5421
<b>mc122</b>	0.2175	0.3162	0.2400	0.2261	0.4662	0.5337	0.4576	0.5424
<b>mc123</b>	0.2219	0.330	0.2229	0.2242	0.4472	0.5527	0.4449	0.5551
<b>mc124</b>	0.2130	0.321	0.2306	0.2350	0.4657	0.5342	0.4437	0.5563
<b>mc211</b>	0.4054	0.148	0.1751	0.2706	0.4458	0.5541	0.5806	0.4419
<b>mc212</b>	0.3920	0.141	0.1852	0.2816	0.4669	0.5330	0.5772	0.4227
<b>mc213</b>	0.3772	0.176	0.1757	0.2709	0.4466	0.5533	0.5530	0.4470
<b>mc214</b>	0.3661	0.167	0.1835	0.2829	0.4665	0.5334	0.5497	0.4503
<b>mc221</b>	0.3515	0.201	0.2317	0.2153	0.4471	0.5528	0.5833	0.4167
<b>mc222</b>	0.3381	0.196	0.2405	0.2252	0.4658	0.5341	0.5786	0.4213
<b>mc223</b>	0.3275	0.225	0.2219	0.2251	0.4471	0.5528	0.5495	0.4505
<b>mc224</b>	0.3141	0.218	0.2327	0.2349	0.4676	0.5323	0.5468	0.4532
<b>mc311</b>	0.5523	0.157	0.0652	0.22	0.2899	0.7100	0.6175	0.3824
<b>mc312</b>	0.5441	0.149	0.0696	0.2368	0.3064	0.6935	0.6138	0.3862
<b>mc313</b>	0.5163	0.185	0.0663	0.2319	0.2983	0.7016	0.5826	0.4173
<b>mc314</b>	0.5080	0.177	0.0712	0.2429	0.3142	0.6857	0.5793	0.4207
<b>mc321</b>	0.5155	0.218	0.0998	0.1659	0.2658	0.7341	0.6154	0.3845
<b>mc322</b>	0.5070	0.211	0.1058	0.1758	0.2816	0.7183	0.6128	0.3871
<b>mc323</b>	0.4818	0.240	0.0942	0.1836	0.2778	0.7221	0.5761	0.4238
<b>mc324</b>	0.4726	0.233	0.1007	0.1931	0.2939	0.7060	0.5734	0.4266
<b>mc411</b>	0.1903	0.163	0.3520	0.2937	0.6457	0.3542	0.5423	0.4576
<b>mc412</b>	0.1773	0.155	0.3607	0.3066	0.6674	0.3325	0.5381	0.4619
<b>mc413</b>	0.1773	0.190	0.3403	0.2919	0.6322	0.3677	0.5176	0.4823
<b>mc414</b>	0.1638	0.181	0.3506	0.3036	0.6543	0.3456	0.5145	0.4854
<b>mc421</b>	0.1430	0.223	0.4017	0.2317	0.6334	0.3665	0.5447	0.4552
<b>mc422</b>	0.1308	0.216	0.4096	0.2432	0.6529	0.3470	0.5405	0.4595
<b>mc423</b>	0.1331	0.244	0.3829	0.2392	0.6221	0.3778	0.5161	0.4839
<b>mc424</b>	0.1201	0.237	0.3909	0.2515	0.6425	0.3574	0.5111	0.4889

## 11 Appendix 3: Data Sources and Constructions

We use data from two sources: a survey about the financing conditions of innovative projects for established manufacturing firms and the Banque de France Balance Sheet Data.

### 11.1 Sources

#### 11.1.1 The FIT survey

We use the survey “Financement de l’Innovation Technologique” (FIT) that was conducted in 2000 by the French Ministry of Industry. Its aim was to obtain statistical information about the financing conditions of innovative projects of manufacturing firms in France. This survey allows to identify the firms which undertook innovative projects between 1997 and 1999 and it gives qualitative information about the financial constraints that firms may have experienced when planning and conducting those projects. A sample of 5500 companies representative of the manufacturing activity (excluding agricultural-food and building sectors) was surveyed. Firms with 20+ employees were surveyed with a response rate of 85%, while every firm with 500+ employees is included in the datasets). It is important to notice that start-ups and new established firms are not in the field of this survey. Globally, the rate of response amounts to 70% (Sessi 2002) so that about 3700 firms are present in the available FIT sample.

As the Community Innovation Surveys (CIS), the FIT survey is based upon the technological innovation concept exposed in the Oslo manual (OECD 1997).<sup>17</sup> This measure of innovative activities is less restrictive than R&D expenditures or patents data and was set up to overcome some shortcomings associated with R&D and patents. For instance, innovative activities are not systematically associated with R&D investments and patents are also strategic tools that are not necessarily used by firms to protect innovation. Moreover, the identification of innovative firms according to the OECD definition grow on practical reasons as we need to observe both the innovative behaviour of the firm and its assessment about financial difficulties.

- **Innovative firms (definition)**

As we are interested in identifying firms with innovative activities (and not only those that succeeded in introducing their innovation on the market), we qualified as “innovative” a firm that have introduced or develop a product or process innovation or

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<sup>17</sup>The Community Innovation Surveys (CIS) are conducted in each country by the national statistical entities in order to collect information about the innovative activities of firms. They are based on the same harmonised questionnaire that may be completed at the national level by additional questions. The survey used here (Financement de l’Innovation Technologique, FIT) is different because it is focused on the financing of innovation. However, its methodological framework is the same as the well-known CIS’ one, in particular concerning the definition of innovation and the structure of the questionnaire.

that have been in process of doing so during the surveyed period. This identification of innovative firms is built on the three following questions:

1) *In 1997, 1998 or 1999, did Your enterprise introduce onto the market any new or significantly improved products for Your enterprise?*

2) *In 1997, 1998 or 1999, did Your enterprise introduce onto the market any new or significantly improved process for Your enterprise?*

3) *In 1997, 1998 or 1999, did Your enterprise have projects of new or significantly improved products or processes:*

- *Which are not yet completed or not yet introduced to the market?*
- *Which were failures?*

In other words, a firm is innovative when it answered positively to at least one of these three questions.

#### • **Financing constraints (definition)**

The qualitative information about the obstacles to innovation is given in the last part of the questionnaire. **All surveyed firms** have to answer the following question:

*In 1997, 1998 or 1999, what are the obstacles that have prevented your firm to conduct or to start innovative projects (multiple answers possible)?*

- *Excessive perceived economic risk*
- *Lack of qualified personnel*
- *Innovation costs too high*
- *Lack of sources of finance*
- *Slowness in the setting up of the financing*
- *Too high interest rates of the financing*
- *Excessive get out clause in the shareholder agreement*
- *Lack of knowledge about ad hoc financial networks*
- *No obstacle*

In addition, the firm has to tick the effect of each listed hampering factors on their innovative projects: (seriously delayed, abandoned or prevented to be started). As a firm may have several innovative projects, it can mention several consequences of obstacles (for instance, both delayed and non started projects).

We consider that a firm faced **financing constraints** when it answered that it has seriously delayed, abandoned or non-started projects because of:

- *Too high interest rates of the financing*
- *Lack of sources of finance*
- *Slowness in the setting up of the financing*

#### • **Subsample of potentially innovative firms**

A significant part of the firms in our initial sample (44%) answered simultaneously that they have not completed nor are in process of implementing innovative projects, and that they do not encounter any obstacle to innovation. Consequently, it could be assessed that this group of firms does not **wish** to innovate and thus, that those firms are not concerned by obstacles to innovation in general and by financial obstacles in

particular. To try to identify the firms that wished to innovate, we define two groups of firms:

The **potentially innovative firms** are the firms that answered positively the first three questions (*i.e.*, firms that introduced or developed a product or process innovation or that were in process of doing so during the surveyed period) **or** the non innovative firms that faced obstacles to innovation. Thus, some of those firms are innovative as defined above (they succeeded in starting, even in completing their projects) while the other ones were not able to start any of their innovative projects.

The second type of firms (the **“others”**) are the non innovative ones that ticked they did not face any obstacle to innovation. Consequently, it may be assessed that these firms did not wish to innovate.

### 11.1.2 The Banque de France Balance Sheet Dataset

In order to have more information about the surveyed firms (their size, economic performance and financing structure), we use the Banque de France Balance Sheet Dataset.<sup>18</sup> This is a database containing essentially very detailed accounting data of French companies, obtained from their fiscal forms plus some complementary questionnaires. The database includes all businesses with more than 500 employees and a fraction of smaller firms so that the member firms amount to around 34,000 companies. It achieves an overall coverage rate of 57% in industry (in terms of number of employees). This rich database is used by the Banque de France to update knowledge of the structure and performance of the French productive system. In addition, it makes it possible for example, to pinpoint sources of financing, to isolate group financing or to identify expenditures in intangible goods and services.

## 11.2 Our cross-section sample

The cross-section sample results from the matching of these two sources. We were able to recover about 60% of the FIT sample companies. After some necessary cleaning, our sample contains 1940 firms.<sup>19</sup> The distribution of the firms in our sample according to their innovative behavior and financing obstacles is given in the table below:

Table 9: Number of firms in the cross-section sample

Potentially innovative firms (1082)				Others
with innovative activities		without innovative activities		
financially constrained	financially unconstrained	financially constrained	financially unconstrained	
198	613	112	159	858

Sources : Centrale de Bilans (Banque de France), FIT (Sessi)

<sup>18</sup>The "Centrale de bilans" dataset.

<sup>19</sup>The manufacture of coke, refined petroleum products and nuclear fuel has been deleted because only two firms were present in the merged dataset. In addition, the firms with negative value added or with abnormally high investment rates have been excluded. This concerns only two firms.

Table 10: Definition of the variables

Name	Definition
Dependent variable : $y_{1i}$	=1 if the firm was innovative, =0 otherwise
Explanatory : $x_{1i}$	
Size	$\log(\text{number of employees})$
Market share	$\frac{\text{sales of the firm}}{\text{sales of the sector}} \times 100$
TP1	=1 if the firm's market is technologically not innovative (reference)
TP2	=1 if the firm's market is weakly innovative,
TP3	=1 if the firm's market is moderately innovative
TP4	=1 if the firm's market is strongly innovative
Financial constraints	=1 if the firm faced financial constraints, =0 otherwise
Financial constraints equation	
Dependent variable : $y_{2i}$	=1 if the firm faced financial constraints, =0 otherwise
Explanatory : $x_{2i}$	
Size	$\log(\text{number of employees})$
Collateral	$\log(\text{tangible assets})$
Banking debt ratio	$\frac{\text{Banking debt}}{(\text{Own financing} + \text{Market Financing} + \text{Financial debt})} \times 100$
Own financing ratio	$\frac{\text{Own financing}}{(\text{Own financing} + \text{Market Financing} + \text{Financial debt})} \times 100$
Gross operating profit margin	$\frac{\text{EBDIT}}{\text{Value added}} \times 100$

Sources : Centrale de Bilans (Banque de France), FIT (Sessi) and EAE (INSEE)

Table 11.a: Descriptive statistics (full sample of 1940 firms)

	Mean	Std	Min	Max
Innovation	0.418	0.493	0	1
Size	4.783	1.107	2.890	9.716
Market share	0.177	0.566	0.001	16.15
TP1	0.139	0.312	0	1
TP2	0.416	0.493	0	1
TP3	0.348	0.476	0	1
TP4	0.097	0.297	0	1
Financial constraints	0.160	0.366	0	1
Collateral	71.048	22.698	4.241	302.444
Banking debt ratio	17.678	15.758	0	92.307
Own financing ratio	31.827	24.195	-609.459	90.136
Gross operating profit margin	18.248	19.416	-197.600	76.850

Sources : Centrale de Bilans (Banque de France), FIT (Sessi) and EAE (INSEE)

Table 11.b: Descriptive statistics (subsample of 1082 potentially innovative firms)

	Mean	Std	Min	Max
Innovation	0.750	0.433	0	1
Size	4.985	1.168	2.890	9.716
Market share	0.208	0.477	0.0011	6.057
TP1	0.050	0.237	0	1
TP2	0.389	0.488	0	1
TP3	0.414	0.493	0	1
TP4	0.147	0.354	0	1
Financial constraints	0.287	0.452	0	1
Collateral	72.062	22.350	10.587	294.207
Banking debt ratio	17.403	15.958	0.000	92.308
Own financing ratio	31.758	19.204	-57.624	90.136
Gross operating profit margin	18.377	19.689	-125.953	72.018

Sources : Centrale de Bilans (Banque de France), FIT (Sessi) and EAE (INSEE)

### 11.3 Our panel sample (1512 firms)

The panel sample is obtained by matching the survey FIT i) with the second French wave of the Community Innovation Survey (CIS2) run by the French Ministry of Industry for Eurostat ii) and with the balance sheet dataset of the Banque de France (Centrale de Bilans). As mentioned before, the survey FIT is based upon the Oslo manual (OECD, 1997) that provides the guidelines for the CIS surveys that is why both in FIT and CIS the same questions are asked to identify innovative firms. Moreover, CIS2, which covers the period 1994-1996, includes similar questions about financial constraints as in FIT (The following wave of the Community Innovation Survey (CIS3) covering 1998-2000 does not include questions about financial constraints therefore we cannot use it here).

The sample obtained by matching FIT, CIS2 and Centrale de Bilans contains 1512 firms (of which 732 “potentially innovative” firms). The transitions for innovation and financial constraints between the two surveyed periods are reported in the tables below.

Table 12.a: Innov Transitions 1994-6 → 1997-9

		1997-1999 (FIT)		
		$I_{it} = 1$	$I_{it} = 0$	Total
1994-1996 (CIS2)	$I_{i,t-1} = 1$	84.45 543 60.54	40.74 354 39.46	42.53 897 100
	$I_{i,t-1} = 0$	15.55 100 16.26	59.26 515 83.74	57.47 615 100
	Total	100 643 42.53	100 869 57.47	100 1512 100

Table 12.b: FinCons Transitions 1994-6 → 1997-9

		1997-1999 (FIT)		
		$F_{it} = 1$	$F_{it} = 0$	Total
1994-1996 (CIS2)	$F_{i,t-1} = 1$	41.32 100 33.00	15.98 203 67.00	20.04 303 100
	$F_{i,t-1} = 0$	58.68 142 67.00	84.02 1067 88.25	76.96 1209 100
	Total	100 242 16.01	100 1072 83.99	100 1512 100

**Legend:**

col %
Cell
Count
row %

Table 12.c: 1994-6 -> 1997-9 Transitions

		1997-1999 (FIT)				
		$I_{it} = 1$ and $F_{it} = 1$	$I_{it} = 1$ and $F_{it} = 0$	$I_{it} = 0$ and $F_{it} = 1$	$I_{it} = 0$ and $F_{it} = 0$	Total
1994-1996 (CIS2)	$I_{i,t-1} = 1$ and $F_{i,t-1} = 1$	15.8 77	37.7 58	14.8 13	7.3 57	13.6 205
		37.6	28.3	6.3	27.8	100
	$I_{i,t-1} = 1$ and $F_{i,t-1} = 0$	68.9 337	46.1 71	30.7 27	32.9 257	45.8 692
		48.7	10.3	3.9	37.1	100
	$I_{i,t-1} = 0$ and $F_{i,t-1} = 1$	1.4 7	7.8 12	19.3 17	7.9 62	6.5 98
		7.1	12.2	17.3	63.3	100
	$I_{i,t-1} = 0$ and $F_{i,t-1} = 0$	13.9 68	8.4 13	35.2 31	51.9 405	34.2 517
		13.2	2.5	6.0	78.3	100
	Total	100 489	100 154	100 88	100 781	100 1512
		32.3	10.2	5.8	51.7	100

**Legend:**

col %
Cell Count
row %