# Self-fulfilling Beliefs and Bounded Bubbles in the U.S. Housing Market*

Masanori Kashiwagi†

February 2010

**Abstract**

This paper provides an equilibrium framework to study the following empirical observations in the U.S. housing market: (i) housing tenure and vacancies are approximately constant, (ii) rents are approximately constant, and (iii) in the late 1990s there was a large house price appreciation. Borrowing ideas from search and matching theory, and closing the model with self-fulfilling beliefs about the housing market, the model generates a house price bubble as a consequence of multiple underlying steady state equilibria. To select an equilibrium, household confidence is assumed to take one of two sunspot-driven values: normal or exuberant. When confidence is normal, both rent and house price are low. When confidence is exuberant, both rent and house price are high. Randomization over these two equilibria implies a substantial increase in house prices as the probability of the exuberant state increases. The model can explain a house price bubble as a rational expectations equilibrium driven by self-fulfilling beliefs.

*I am truly indebted to Roger Farmer for his supervision and encouragement. I am also grateful to Lee Ohanian and Pierre-Olivier Weill for fruitful discussions. For helpful comments, I wish to thank Andy Atkeson, Paco Buera, Ariel Burstein, Matthias Doepke, Christian Hellwig, Thomas Hintermaier, Venky Venkateswaran, Mark Wright and seminar participants at UCLA and the Development Bank of Japan. I gratefully acknowledge financial support from UCLA Ziman Center for Real Estate.

†UCLA Department of Economics, Email: mkashiwa@ucla.edu

# 1 Introduction

This paper studies the recent boom in U.S. house prices as a sunspot phenomenon in a rational expectations equilibrium framework. Empirical data suggest that the U.S. housing market is stationary in terms of demographics and quantities. Among the total housing units, the shares of rented, owned and vacant units are stable over time. By contrast, house prices grew rapidly from the late 1990s until 2006, while rent was stable. Applying ideas developed in search and matching theory, this paper presents a simple model that accounts for these facts.

In the search and matching model, the meeting of traders creates a surplus which must be divided between them. As Howitt and McAfee (1987) point out, this leads to a situation with fewer equations than unknowns. Consequently, the model displays a steady state indeterminacy. Most literature resolves this indeterminacy by assuming Nash bargaining with a fixed bargaining weight. This paper, however, takes a different route. As suggested by Farmer (2009), I close the model by treating confidence as a fundamental. To select an equilibrium, I assume that household confidence may take one of two values: normal or exuberant. When confidence is normal, both rent and house price are low. When confidence is exuberant, both rent and house price are high. I assume that confidence is driven by a sunspot as in Cass and Shell (1983). Randomization over these two equilibria implies a substantial increase in house prices as the probability of the exuberant state increases. The model can explain a house price bubble as a rational expectations equilibrium driven by self-fulfilling beliefs.

Sunspots work as a signal that generates the correct probabilities, and people coordinate on that signal as a way of moving from one equilibrium to another. While the economy is likely to be in a normal state most of the time, the random arrival of news may signal the possibility that an exuberant state is likely to occur. After receiving such news, people get incremental evidence about the likelihood that it is true. As news arrives randomly, it progressively drives the economy towards the exuberant state. Alternatively, households may receive a signal that the news was incorrect, which triggers a collapse back to the normal state.

House prices increase as the economy moves closer to the exuberant economy, while rent is stable along the path. In this framework, news drives prices. This paper approaches the U.S. experience of housing market bubbles within this conceptual framework. Unlike the standard rational bubbles argument, variations in house prices are bounded as traders appropriate positive surplus. Accordingly, I call the phenomena "bounded bubbles." A prominent feature of this paper is that it provides an equilibrium framework to support a nonfundamental account of the recent housing boom and collapse.

The large appreciation of house prices was possibly due to an increase in real income associated with economic growth. An increase in household income leads to more spending on housing services. To control for this explanation, I deflate nominal prices by nominal income. The proposed house price series still exhibits the surge from the late 1990s, but the rent series is stable.

Section 2 of this paper provides a literature review. Section 3 documents three observations regarding the U.S. housing market from 1975 to 2007/2008: (i) housing tenure and vacancies were approximately constant, (ii) rents were approximately constant, and (iii) during the late 1990s, there was a large appreciation of house prices. Section 4 presents a theoretical model. Section 5 discusses quantitative results and shows that recent housing bubbles can be characterized as a rational expectations equilibrium. Section 6 concludes.

# 2 Related Literature

The model in this paper applies ideas developed in the labor search literature (for example, see Mortensen and Pissarides (1994)) to the housing market. The pioneering work by Wheaton (1990) studied the homeownership market and presented comparative statics. Subsequent work taking this approach includes Williams (1995) and Krainer (2001).

Among recent literature that studies the empirical implications of this model, this paper's focus is related to Piazzesi and Schneider (2009), who use a search model to examine the role of a small number of optimistic traders on house prices. They model the surge in house prices as a one-time shock to beliefs of a small fraction of households and present the transition of prices back to the original steady state. This paper instead studies the boom path of prices and stable rent. Among other recent studies, Ngai and Tenreyro (2009) account for seasonal fluctuations in the housing market through a stochastic job matching model due to Jovanovic (1979).

My approach to the housing bubble aligns with Shiller (2007), who argues that it does not appear possible to account for the recent house price boom in terms of fundamentals such as rents and construction costs; instead, the boom operates as a speculative bubble driven largely by extravagant expectations for future price appreciations. Unlike Shiller (2007), this paper characterizes the boom in house prices as a rational expectations sunspot equilibrium.[1]

The notion of sunspot equilibria is taken from the work by Azariadis (1981), Cass and Shell (1983), Azariadis and Guenerie (1986) and Weil (1987). The sunspot equilibria are constructed by randomizing a finite set of steady state equilibria.[2] Assuming that agents share common beliefs about the sunspot activity and coordinate according to those beliefs, sunspots work as a way of moving from one equilibrium to another. Lastly, the idea of sunspots affecting a search economy is related to Farmer (2009, 2010).

# 3 Observations in the U.S. Housing Market

This section presents empirical observations in the U.S. housing market. Using the data between 1975 and 2007/2008, I point out the following three aspects: (i) housing tenure and vacancies were approximately constant, (ii) rents were approximately constant, and (iii) during the late 1990s, there was a large appreciation of house prices. This section discusses the observations of real interest rate and housing starts as additional evidence.

## 3.1 Stability in the Housing Market

I examine U.S. housing stock between 1975 and 2008. Figure 1 presents how housing stock was occupied. It shows the shares of rented, owned, and vacant units among the total housing units on the market.[3] The data are quarterly and taken from the U.S. Census Bureau.

[1] Peterson (2009) also uses a search model and presents a framework complementary to Shiller's argument. In Peterson (2009), households ignore the effects of search frictions on past prices and think that there has been a permanent change in the value of a house.

[2] As shown in Cass and Shell (1983) and Azariadis and Guesnerie (1986), multiplicity of certainty equilibria is not necessary for the existence of a sunspot equilibrium.

[3] I excluded housing units for occasional use and those occupied by people who usually live elsewhere.

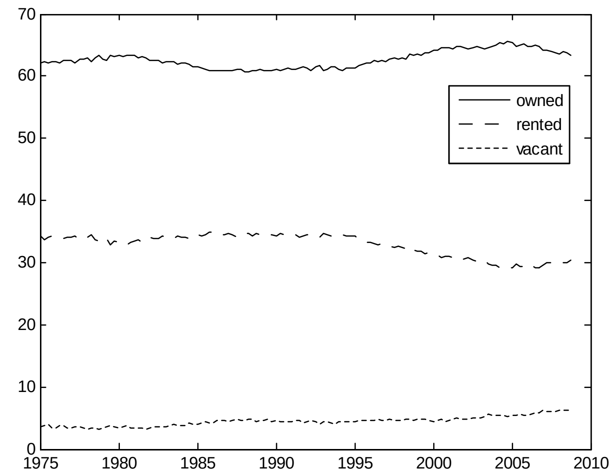


Figure 1: Occupancy of Housing Units (percent)

The proportions remain approximately constant over the sample period. More than 60 percent of all the housing units in the nation are occupied by owners. About 35 percent are rented, and the rest (about four percent) are vacant. Though these series include small variations, I take them as flat lines. As I present below, house prices drastically increased despite the stability of housing occupancy.

In the census data presented, the count of occupied housing units is the same as the count of households.[4] Figure 1 implies that the number of total housing units per household is also constant over time. In other words, the number of households and total housing units grow almost at the same rate.

[4]See U.S. Census Bureau, Housing Vacancies and Homeownership: Definitions (http://www.census.gov/hhes/www/housing/hvs/hvs.html).

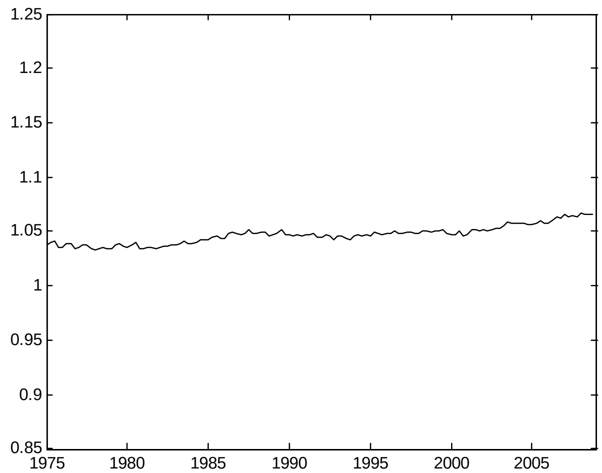


Figure 2: Total Housing Units per Household

Figure 2 plots the number of total housing units per household. Over the sample period, there are 4-5 percent more housing units than the total number of households. This figure implies that, proportional to the population, quantities in the housing market are approximately constant. Population growth in the U.S. leads to more households, which expands housing demand. Controlling this effect by deflating the number of households leads to stable housing stock in the market. In terms of demographics, therefore, house prices should present stable behavior. Nonetheless, this was not the case during the sample period.

## 3.2 House Prices

I use house price indices published by the Federal Housing Finance Board (FHFB).[5] These measure changes in single-family house prices. They are repeat-sales indices, that is, they measure average price changes of housing properties that are sold at least twice. Measuring only repeat transactions on the same housing units helps control for changes in housing units' quality. Consequently, the indices show quality-adjusted house prices.

To construct the time series of nominal house prices, I combine the level information about the house prices with FHFB house price indices. I use the nominal median home values reported in Davis *et al.* (2008). Figure 3 plots the quarterly time series of U.S. real house prices, with the first observation (1975Q1) normalized to one. To construct

[5] Previously, the Office of Federal Housing Enterprise Oversight.

the series of real house prices, I take the national consumer price indices (CPI) for commodities less shelter from the Bureau of Labor Statistics (BLS), following Davis *et al.* (2008).

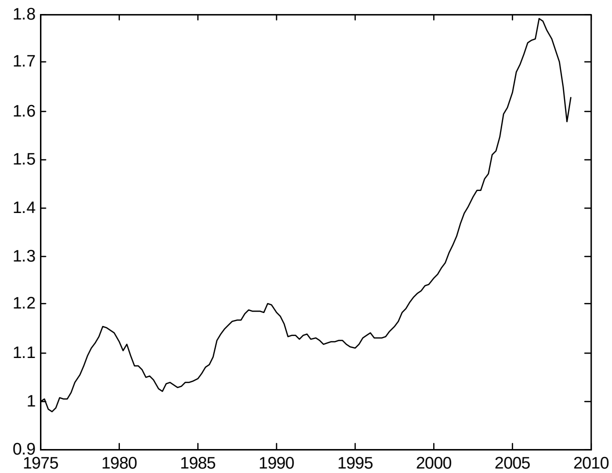


Figure 3: Real House Prices

There are some striking features in the behavior of real prices of U.S. houses. From the mid 1990s, house prices increased rapidly, peaking in the fourth quarter of 2006. Compared to values in 1995, they increased 62 percent by 2006Q4. After the 2006Q4 peak, prices fell. The price slide over the next few quarters, however, was not as drastic as the plunge in 2008. Between 2007Q3 and 2008Q3, national house prices fell 9.9 percent. Before the mid-1990s, national house prices were stationary.

To further examine the house price series, I deflate the nominal house price series by nominal income. The nominal income series contains information on inflation and growth. It is important for house prices because people with higher incomes are willing to purchase more expensive houses. Economic growth reflected in changes in income may account for the rise in house prices. I take the median nominal income series from the U.S. Census Bureau. I use the national data from 1975 to 2007, and the series is annual. Figure 4 plots this "house price in income units" series annually from 1975 to 2007. The nominal house prices are computed as annual averages from the original quarterly series.

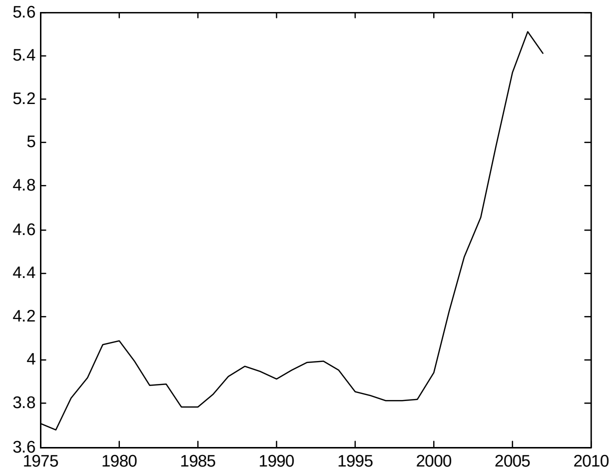


Figure 4: Nominal House Prices Deflated by Nominal Income

Inflation and growth do not fully account for the recent surge in house prices.[6] The overall picture is similar to the real house price series in Figure 3. We see fairly stable behavior before the boom and then a rapid increase in the series. House prices in income units started to grow later than real house prices; they surged during the late 1990s and kept increasing until 2006.

To compare the two house price series, I rescale them by normalizing the first observation (1975) to one. Figure 5 plots the rescaled series. The real price series is annualized by taking averages and rescaled in the same way as the house price in income units series. The figure shows that deflating by nominal income emphasizes both the price stability that existed until the late 1990s and the subsequent boom. The stability before the boom is consistent with the stationary behavior of quantities noted above.

[6]It may be argued that the price increases are due to rises in the average house size because the repeated house price index does not control for trends in the quantity of housing services. To control for the increase in average house size, Van Nieuwerburgh and Weill (2009) deflate the real house price series by the average growth rate of house size between 1975 and 2007. The resulting series still exhibits a substantial increase in house prices.

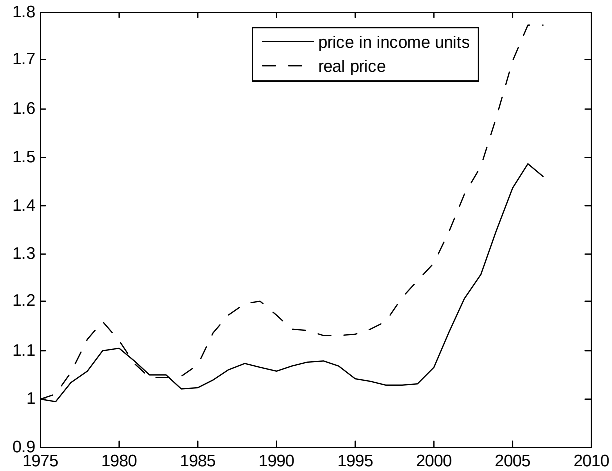


Figure 5: Two House Price Series Compared

### 3.3 Rents

While house prices experienced a very long increase, the rent series remained very stable. Taking the level information of annual rent from Davis *et al.* (2008), I combine the information with the CPI for rent published by BLS. Figure 6 presents the series of nominal rent deflated by nominal income with the first observation normalized to one. Like the house occupancy series, the rent series is approximately constant over the sample period.

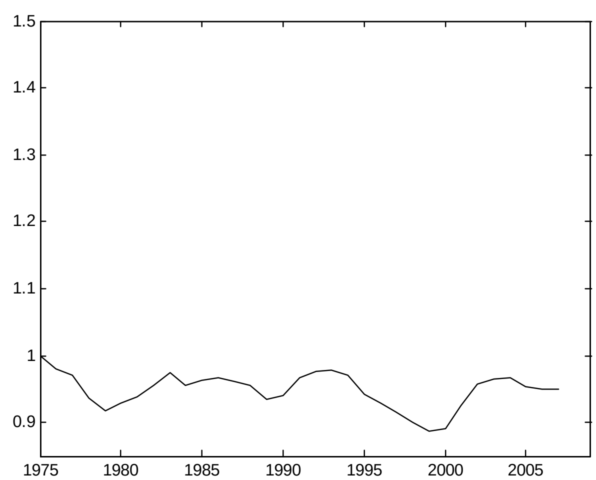


Figure 6: Rents in Income Units

In summary, we observe the following over the sample period: Quantities in the housing market are stationary, in that occupancy of the housing units is stable. The respective shares of rented, owned and vacant units to the total housing stock remain approximately constant. The numbers of households and housing units grow approximately at the same rate. And the rent series is stable, with only small fluctuations.

Nonetheless, house prices appreciated substantially. This paper provides a search and matching model and characterizes those observations as a rational expectations sunspot equilibrium by allowing multiplicity of deterministic equilibria.

## 3.4 Other Data

An obvious candidate to explain an asset price boom is a change in the interest rate. Suppose that there is no growth, time dependence or uncertainty. If, for example, the real interest rate drops from six percent to four percent, asset prices will be about 50 percent higher.[7]

To explore this possibility, I plot the real interest series computed by using the 30-year mortgage interest rate[8] and the inflation rate from CPI. Figure 7 shows the results.

We observe a substantial decline in the real interest rate during the early 1980s, and subsequently a slightly downward trend. If the interest rate is a key determinant of house prices, house prices should increase from the 1980s. The historical data, however, does not support such a finding. As already discussed, house prices were stable until late the 1990s, when the boom started.

[7] Let $p$ and $d$ be real asset prices and dividends, respectively. Then we have $p = d\left(1 + \frac{1}{r}\right)$, where $r$ is the real interest rate.

[8] Van Nieuwerburgh and Weill (2009) target the average real 30-year fixed rate mortgage interest rate in calibrating the time discount factor. They argue that it is the most relevant interest rate for computing house prices when using the present value formula.

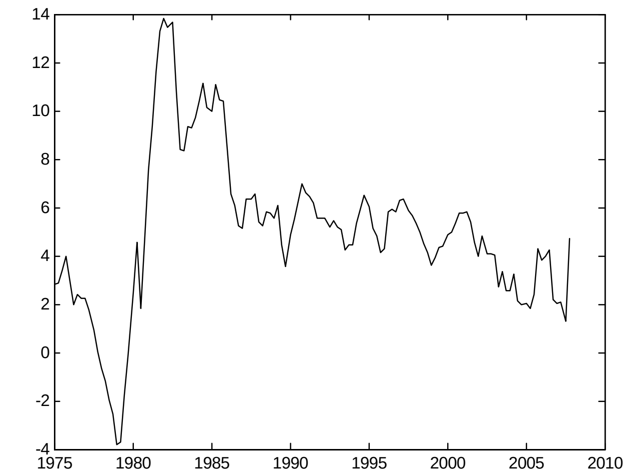


Figure 7: Real Interest Rate

Housing starts do not seem be important in determining prices either. Figure 8 presents the time series of housing starts deflated by the number of households, to control for population growth. Housing starts per household experienced large swings between 1975 and the early 1990s, when house prices were stable. During the surge in house prices, the series does not increase as much as the earlier peaks.

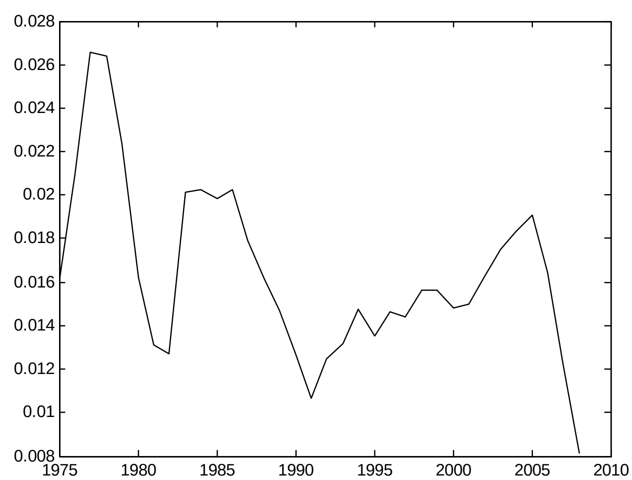


Figure 8: Housing Starts Deflated by Total Number of Households

# 4 Model

This section presents a search and matching model of the housing market. Time is discrete and runs forever. There are two groups of people: renters and homeowners. On the supply side, there are competitive real estate agencies that allocate houses for rent or for sale. In the ownership market, agents have to go through a search and matching process.

The model has many features of the standard search and matching framework, but the assumption of a fixed bargaining weight is relaxed. Then there are multiple underlying steady state equilibria. As a consequence, there can be a rational expectations equilibrium with a house price bubble.

## 4.1 Households

I assume that the model economy has a fixed measure of households. Households are risk neutral and discount future utility at a constant rate $\beta$. Each household either rents or owns one housing unit. I assume that households draw higher utility from owning units than from renting units because of tax benefits and psychological satisfaction (for example, pride and sense of security). Consequently, renters seek houses to purchase. The probability of finding a housing unit to buy is subject to the search and matching friction described below. I assume an endowment economy, and in each period households receive a flow of income. Let $R_t$ be the value of being a renter at period $t$. Then the value satisfies the following Bellman equation:

$$R_t = w + \alpha_R - q_t + \beta E_t \left[ \phi_{t+1} \left( H_{t+1} - p_{t+1} \right) + \left( 1 - \phi_{t+1} \right) R_{t+1} \right], \tag{1}$$

where $w$ and $\alpha_R$ are a flow income and utility from renting a housing unit, respectively. In period $t$, renters pay rent $q_t$, and in the following period they search for homeownership. With probability $\phi_{t+1}$, they find and purchase a housing unit at price $p_{t+1}$ and then become homeowners. The value of being a homeowner in period $t$ is denoted by $H_t$. If searching renters do not find a house to buy, with probability $1 - \phi_{t+1}$, they remain renters.

Homeowners receive the same income flow $w$ as renters. I assume that they enjoy utility $\alpha_H > \alpha_R$ from their own homes. Unlike renters, homeowners have no rent payment. In the following period, homeowners may have to move out of their housing units due to exogenous events with probability $m_H$. (Such an event might be, for example, a job reassignment to another location.) In these cases, I assume that homeowners sell their housing units to the real estate sector and change their housing tenure into renters. This assumption simplifies the framework because one does not have to keep track of households' number of housing units. In addition, this assumption is without loss of generality because the real estate sector is implicitly owned by households in the model.

The real estate sector represents the opportunity cost of holding a vacant unit and trying to sell it in the ownership market at a higher price. Here, I abstract from agents who move from an owned unit to another owned unit. This approach differs from Wheaton's (1990) framework, which focuses on the homeownership market. According to the American Housing Survey data, however, about half of homeowners who move become renters.

The present value of a vacant housing unit at time $t$ is denoted by $W_t$. I assume that the real estate sector is competitive. Hence, when homeowners move out, their housing units are sold to the real estate sector for the value of vacancy $W_t$, and the homeowners become renters. The value of being a homeowner is described recursively as:

$$H_t = w + \alpha_H + \beta E_t \left[ m_H \left( R_{t+1} + W_{t+1} \right) + \left( 1 - m_H \right) H_{t+1} \right]. \tag{2}$$

## 4.2 Real Estate Sector

I assume a representative real estate sector that supplies housing units to the market. As mentioned above, if homeowners move out of their housing units, the real estate sector purchases those units for the present value of a vacant unit $W_t$ due to competitiveness. The real estate sector has two options to put housing units on the market. First, it can rent out houses, in which case they are always occupied by renters and the real estate sector receives rent payment $q_t$ in period $t$. Second, it can post housing units for sale to potential new homeowners.[9] In such cases, the real estate sector finds a new homeowner with some probability, selling the housing unit at price price $p_t$ during period $t$. This option is subject to search and matching friction.

Let $W_{FR,t}$ be the value of the first option, renting out a housing unit for period $t$. Similarly, the value of the second option, posting a housing unit for sale, is denoted by

[9] The real estate sector serves as an intermediary in the model economy. Rubinstein and Wolinsky (1987) analyze the activity of intermediaries in bilateral trading. In their model, buyers and sellers can trade directly or indirectly through the intermediaries while here agents trade only through the real estate sector.

$W_{FS,t}$. The value of renting out today is described as:

$$W_{FR,t} = q_t + \beta E_t \left[ \max \left\{ W_{FR,t+1}, W_{FS,t+1} \right\} \right],$$

that is, the real estate sector receives rent payment for certain today and in the next period it faces the two options again.

If the real estate sector chooses to post a housing unit for sale, it faces search and matching friction. Let $\theta_t$ denote the probability that the real estate sector finds a new homeowner to purchase the housing unit. Then, $W_{FS,t}$, the value of posting a housing unit for sale satisfies the following Bellman equation:

$$W_{FS,t} = \theta_t p_t + (1 - \theta_t) \beta E_t \left[ \max \left\{ W_{FR,t+1}, W_{FS,t+1} \right\} \right].$$

If the real estate sector successfully matches with a renter, with probability $\theta_t$, transaction occurs at price $p_t$. Otherwise, the real estate sector holds on to the housing unit and can choose between the two options in the following period.

Since the real estate sector is assumed to be competitive, the values of renting out and posting for sale equal the value of a vacant housing unit. This implies that

$$W_t = W_{FS,t} = W_{FR,t},$$

for all $t$. Hence, the real estate sector is characterized by zero profit. (Recall that the real estate sector buys housing units for $W_t$ from homeowners.)

This condition attains the interior solution due to the indifference of renting out a house to posting it for sale. If the value of posting housing units for sale is higher than the other option ($W_{FS,t} > W_{FR,t}$), the real estate sector would put every housing unit in the homeownership market; as a result, there would be no housing units for renters. In the reverse case ($W_{FS,t} < W_{FR,t}$), there would be more rental units than renters. There is no friction in the rental market in that renters can find housing units to rent instantly; this pushes rents down until the value of renting out equals that of selling. This indifference condition guarantees that these two corners are avoided in equilibrium. Taken together, the equilibrium equations that characterize the real estate sector's behavior are given by:

$$W_t = q_t + \beta E_t \left[ W_{t+1} \right], \tag{3}$$

and

$$W_t = \theta_t p_t + (1 - \theta_t) \beta E_t [W_{t+1}] . \tag{4}$$

Equation (3) implies that the value of a vacant housing unit is the present discounted sum of flow rents. According to equation (4), there is a "liquidity premium" in that the transaction price in the ownership market is higher than the value of a vacant unit that can be rented out for certain. The premium is characterized by the search friction and the discount rate. In the steady state, the transaction price $p$ and the value of a vacant unit $W$ satisfy $p = \frac{1-(1-\theta)\beta}{\theta} W$ and the coefficient is greater than one if $0 < \theta < 1$. The premium compensates for the risk that the real estate sector will fail to find a new homeowner and holds on to a housing unit as vacancy until the next period.

## 4.3 Evolution of Housing Inventory

This subsection describes how the measures of renters and homeowners evolve in the model over time. Let $\mu_{R,t}$ and $\mu_{H,t}$ be the measure of renters and homeowners at the beginning of period $t$, respectively. As I assume that each household either rents or owns one housing unit, these variables also indicate the respective measures of rented and owned units. I consider a stationary model by abstracting from population growth and increases in housing units. The total number of households is denoted by $\bar{N}$, which implies that

$$\mu_{R,t} + \mu_{H,t} = \bar{N},$$

for all $t$. I also assume that the total number of housing units is exogenous and constant. Let $\mu_{V,t}$ denote the stock of vacant housing units at the beginning of period $t$. As all the housing units are rented, owned or vacant, we have

$$\mu_{R,t} + \mu_{H,t} + \mu_{V,t} = \bar{H},$$

for all $t$, where $\bar{H}$ denotes the total measure of housing units and is constant. In this framework, the measure of vacant units is simply given by $\mu_{V,t} = \bar{H} - \bar{N}$. This model abstracts from fluctuations in vacant units, based on the observations that the numbers of households and housing units grew at approximately the same rate and that the rise in housing starts was small during the boom period relative to earlier times. Carefully examining this margin is interesting.[10] I, however, take the opposite approach of fixed housing units in the model economy.

Renters seek housing units to purchase but face search and matching friction. Let $M_t$ denote the measure of new matches between renters and housing units posted for sale. Among the stock of renters, measure $M_t$ of them purchase houses and become homeowners. At the same time, there is an inflow of renters, as homeowners who move sell their units to the real estate sector and become renters. The probability of drawing a moving shock is $m_H$. Hence, the stock of renters in the following period is characterized by the law of motion of renters:

$$\mu_{R,t+1} = \mu_{R,t} - M_t + m_H \mu_{H,t}, \tag{5}$$

for all $t$. Accordingly, the new flow of homeowners is given by the new match, $M_t$. The outflow is those who are hit by the moving shock. The law of motion of homeowners is given by:

$$\mu_{H,t+1} = \mu_{H,t} + M_t - m_H \mu_{H,t}, \tag{6}$$

for all $t$. This clearly holds from the law of motion of renters (equation (5)) and the fixed measure of households.

## 4.4 Matching

The measure of new matches, $M_t$, is a function of the measure of households searching for housing units to purchase and the measure of housing units that the real estate sector posts for sale. As every renter searches for a housing unit in the model, the first element is the measure of renters, $\mu_{R,t}$. Let $\mu_{FS,t}$ be the measure of housing units posted for sale. As in the bulk of the labor search literature, the matching function is assumed to be constant returns to scale and increasing in both arguments. The measure of new matches is given by

$$M_t = M\left(\mu_{R,t}, \mu_{FS,t}\right).$$

After the moving shock is realized, the real estate sector adds the new flow of houses from homeowners who move out to its stock of houses, rental and vacant units.

[10] For example, Kiyotaki *et al.* (2008) present a model in which housing units are produced from capital and fixed land.

In period $t$, the new flow is $m_H \mu_{H,t}$ and the existing stock of houses in the real estate sector is the rental and vacant units, $\mu_{R,t} + \mu_{V,t}$. The real estate sector allocates these housing units between those marketed for rent and those marketed for sale. Let $\mu_{FR,t}$ be the measure of housing units to be rented out. The measure of houses posted for sale is denoted by $\mu_{FS,t}$. Accordingly, the resource constraint on housing stock is

$$\mu_{FS,t} + \mu_{FR,t} = \mu_{R,t} + \mu_{V,t} + m_H \mu_{H,t}. \tag{7}$$

As I assume no friction in the rental market, the real estate sector should supply the same rental units as renters to clear the market. The measure of renters at the beginning of period $t+1$ is denoted by $\mu_{R,t+1}$. Recall that this is a state variable in period $t+1$ and determined within period $t$. Hence the real estate sector rents out the measure $\mu_{R,t+1}$ of housing units to clear the rental market. Therefore, the measure of housing units to be rented out $\mu_{FR,t}$ satisfies:

$$\mu_{FR,t} = \mu_{R,t+1}. \tag{8}$$

The law of motion of renters, equation (5), implies that

$$\mu_{FS,t} - M_t = \mu_{V,t+1}.$$

The stock of vacant units at the beginning of each period equals the measure of housing units that are posted for sale but failed to match with renters.[11]

The measure of matches depends on the measure of renters $\mu_{R,t}$ and the measure of housing units posted for sale $\mu_{FS,t}$. The measure of matches divided by the measure of renters is the probability that a renter succeeds in finding a house, $\phi$. Similarly, the measure of matches divided by the measure of houses posted for sale is the probability that a housing unit on the homeownership market is occupied, $\theta$. Hence, those probabilities are determined through the evolution of the measure of renters and houses posted for sale:

$$\phi_t = \frac{M_t}{\mu_{R,t}}, \ \theta_t = \frac{M_t}{\mu_{FS,t}},$$

for all $t$.

[11] The time subscript is for purposes of correct interpretation. For derivation, the time subscript on the measure of vacant units can be ignored because it is constant.

## 4.5 Rational Expectations Sunspot Equilibrium

While most literature assumes Nash bargaining between traders with a fixed bargaining weight, this paper's model is closed with self-fulfilling beliefs about the housing market. This approach exploits the view that the model exhibits multiple underlying steady state equilibria, as Howitt and McAfee (1987) point out. Consequently, there can be a house price bubble characterized as a rational expectations sunspot equilibrium.

To select an equilibrium, I assume that household confidence may take one of two values: normal or exuberant. When confidence is normal, rent is low and house price is low. When confidence is exuberant, rent is high and house price is high. The normal rent is denoted by $\underline{q}$, and the exuberant rent is denoted by $\bar{q}$. This household confidence defines two underlying steady state equilibria. I consider randomization over those equilibria by news as a sunspot. In the model, quantities are the same for each steady state and accordingly quantities are constant in a randomized equilibrium.

News is coming all the time about whether the exuberant economy may be happening or not. States of the economy are associated with the likelihood of the exuberant economy occurring. The news works as a sunspot and drives the economy from one state to another. The sunspot-driven states are denoted by $s_t$. Hence a variable $X_t$ is a function of $s_t$ and denoted by $X_t(s_t)$.[12]

I assume that there are $N$ states and define the first and $N$th states, respectively, as the exuberant state and the normal state. The other states are named news states because they represent news about the likelihood of the exuberant state happening. Rents are normal in all the states but the exuberant state. Hence rents in this model are

$$q_t(s_t) = \bar{q} \text{ if } s_t = 1 \text{ and } q_t(s_t) = \underline{q} \text{ if } s_t = 2, \dots, N. \tag{9}$$

Assuming the evolution of sunspots is Markovian with transition probability matrix $\Pi$, house prices are endogenously determined for each state. The price and rent in each state are:

$$\underbrace{(p(1), \bar{q})}_{\text{exuberant state}}, \underbrace{(p(2), \underline{q}), \dots, (p(N-1), \underline{q})}_{\text{news states}}, \underbrace{(p(N), \underline{q})}_{\text{normal state}}.$$

The self-fulfilling property of the rational expectations equilibrium implies that prices change because people believe that they will, due to sunspot activity. This exploits the idea of market psychology affecting house prices.

[12] State variables $\mu_{R,t+1}, \mu_{H,t+1}$ and $\mu_{V,t+1}$ are respectively denoted by $\mu_{R,t+1}(s_t), \mu_{H,t+1}(s_t)$ and $\mu_{V,t+1}(s_t)$.

I assume that the transition probability matrix $\Pi$ takes the following structure:

$$\Pi = \begin{bmatrix} 1-\pi_{1,N} & 0 & \cdots & 0 & \pi_{1,N} \\ 1-\pi & 0 & \cdots & 0 & \pi \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & 1-\pi & 0 & \pi \\ 0 & \cdots & 0 & 1-\pi_{N,N} & \pi_{N,N} \end{bmatrix},$$

where the rows and columns represent current and next states, respectively. I define the $N$th state as the normal state and assume that it is very stable in that the probability of remaining there ($\pi_{N,N}$) is very high. At the normal state, however, there is a small probability of moving to the $(N-1)$th state. Once this shock is drawn, there is a fixed probability $1-\pi$ of moving up by one state. With probability $\pi$, the economy goes back to the normal state. This structure allows easy computation of the expected duration of booms conditional on the economy in the $(N-1)$th state. In the later part, I parameterize the probabilities by targeting an expected duration of a boom. The constant probability assumption is not crucial for the computation of duration. One can assume different probabilities of moving up by one state for different states, but the assumption that in news states the economy will move up by one state or return to the normal state is important for simplifying the algorithm.[13]

In this model, shifts in states generate a house price appreciation. On the other hand, those shifts have no feedback on the measures of rented, owned or vacant units. This is due to the assumption that housing units are fixed in the economy and that exogenous moving shocks govern the evolution of renters and homeowners. I focus on balanced flow of renters and homeowners from the observation discussed in Section 3. Then for each state, quantities are the same.

A rational expectations equilibrium must satisfy the individual rationality condition that each trader in the ownership market appropriates positive surplus from a match.

[13] Suppose more generally, the probability of returning to the normal state is different for different states. Let the probability of returning to the normal state if the economy is in state $s$ be $\pi_{s,N}$. As I assume that at intermediate states, the economy either moves up by one state or collapses to the normal state, the expected duration conditional on the economy is in the $(N-1)$th state is

$$D = \pi_{N-1,N} + \sum_{i=2}^{N-2} \left[ i \times \pi_{N-i,N} \left( \prod_{j=N-i+1}^{N-1} (1-\pi_{j,N}) \right) \right] + \left( N - 1 + \frac{1-\pi_{1,N}}{\pi_{1,N}} \right) \prod_{j=2}^{N-1} (1-\pi_{j,N}).$$

For each period $t$, the surplus of a match is the difference in values of being a homeowner and a renter less the value of a vacant unit, $H_t(s_t) - R_t(s_t) - W_t(s_t)$. Then a house buyer receives the surplus of $H_t(s_t) - R_t(s_t) - p_t(s_t)$ from a transaction with price $p_t(s_t)$ and a seller appropriates the rest. Hence individual rationality implies that prices are subject to $H_t(s_t) - R_t(s_t) > p_t(s_t) > W_t(s_t)$. Accordingly, a rational expectations sunspot equilibrium is defined as follows.

**Definition**. A rational expectations sunspot equilibrium of this economy is a sequence of measures of renters, homeowners and vacant units $\left\{\mu_{R,t+1}(s_t), \mu_{H,t+1}(s_t), \mu_{V,t+1}(s_t)\right\}$ with their initial steady state values, the measure of housing units rented out and posted for sale $\left\{\mu_{FR,t}(s_t), \mu_{FS,t}(s_t)\right\}$, the values of being a renter and a homeowner, $\{R_t(s_t), H_t(s_t)\}$, house prices and rent $\{p_t(s_t), q_t(s_t)\}$, and the present value of a vacant housing unit $\{W_t(s_t)\}$. These variables are subject to the Bellman equations (1) and (2), pricing equations (3) and (4), laws of motion (5) and (6), the resource constraint of housing units (7), the rent market clearing condition (8), rents defined by confidence (9), vacancy equation $\mu_{V,t+1}(s_t) = \bar{H} - \bar{N}$, sunspot events $\{s_t\}$ evolving according to the transition probability matrix $\Pi$ and the individual rationality condition $H_t(s_t) - R_t(s_t) > p_t(s_t) > W_t(s_t)$.

## 4.6 Isomorphism

If the model is closed with a Nash bargaining, prices are solved given the Nash bargaining weight. Let $\lambda_t$ be the bargaining weight of a renter. In this approach, prices are formed through maximizing the Nash product $(H_t - R_t - p_t)^{\lambda_t}(p_t - W_t)^{1-\lambda_t}$ and the resulting surplus sharing rule $\frac{\lambda_t}{H_t - R_t - p_t} = \frac{1-\lambda_t}{p_t - W_t}$ works as a condition to solve for prices. Hence, using the model closed with confidence, the implied surplus sharing rate can be computed for state $s_t$ by $\lambda_t(s_t) = \frac{H_t(s_t) - R_t(s_t) - p_t(s_t)}{H_t(s_t) - R_t(s_t) - W_t(s_t)}$. With the implied sharing rate for each state and the transition probability matrix, the model closed with the Nash bargaining implies the same rents and prices. In this sense, the model closed with beliefs is isomorphic to the bargaining framework. Moreover, the individual rationality condition is equivalent to the notion that the implied surplus sharing rate $\lambda_t$ is in an open interval $(0, 1)$.

# 5 Quantitative Analysis

This section discusses some quantitative implications of the model and shows that a house price bubble can be characterized as a rational expectations equilibrium. With the calibrated parameters presented below, the model generates a house price bubble as a consequence of multiple underlying steady state equilibria. Sensitivity analysis suggests that variations of rent and sustaining probability in the exuberant state still imply a considerable increase in house prices.

## 5.1 Parameterization

I calibrate the fixed parameters to a deterministic steady state that corresponds to the period of stability in the U.S. house prices. First, the moving probability of homeowners $m_H$ is calibrated to 0.04, which is the mean ratio of homeowners who become renters to the total homeowners, according to the American Housing Survey. I normalize the total measure of households to be one, $\bar{N} = 1$. The total housing units relative to the number of households $\bar{H}$ is taken from the U.S. Census Bureau. From the total housing units in the data, I subtract vacant units occupied by people who usually live elsewhere, units for temporary use and seasonally vacant units. In other words, the vacant units in the model $\mu_V$ correspond to vacant units on the market. The measure of total housing units is 1.045 per household.

I assume that the matching technology takes a Cobb-Douglas form[14]

$$M\left(\mu_{R,t}, \mu_{FS,t}\right) = \kappa \left(\mu_{R,t}\right)^{\gamma} \left(\mu_{FS,t}\right)^{1-\gamma}.$$

To calibrate the parameters on the matching function, I target the homeownership rate, which is stable around 65 percent. The curvature parameter $\gamma$ is set to be 0.5.[15] Given this curvature parameter, targeting the homeownership rate $\mu_H$ to be 65 percent implies that $\kappa$ is 0.144. Note that the choice of $\gamma$ by itself is not important for the main analysis, as long as the scale parameter $\kappa$ is properly specified to match the observed homeownership rate; this is so because the relevant probabilities $\phi$ and $\theta$ in the ownership market are determined by the steady state homeownership rate. Those

[14] This specification potentially leads to a situation in which $M_t > \min\left\{\mu_{R,t}, \mu_{FS,t}\right\}$ while it is not the case with the parameterization in the quantitative experiment.

[15] The choice of the curvature parameter does not have welfare consequences because the model assumes fixed housing units.

probabilities are computed as $\phi = 0.074$ and $\theta = 0.37$. The rate that renters become homeowners is quite consistent with the data on tenure change. According to the American Housing Survey data, the rate is about seven percent.

Data on the rent-price ratio are used for calibrating the time discount rate. The series is very stable around five percent before the housing price boom. Assuming that this is the steady state value of the rent-price ratio, the discount rate $\beta$ can be calibrated as 0.945.

To calibrate the parameter of flow utility from a housing unit, I use observed rent. Assuming that surplus is equally split in the targeted steady state, then rent is proportional to the difference in the flow utility from housing units, $\alpha_H - \alpha_R$. Nominal rent deflated by nominal income is stable around 0.196, which is the average value between 1975 and 1998. I normalize the flow utility from a rental unit $\alpha_R$ to zero. And the flow utility from homeownership $\alpha_H$ is calibrated as 0.0866. Note that to be compatible with prices in income units presented in Section 3, I normalize the flow income $w$ to one. Hence, homeowners enjoy extra utility equivalent to 8.66 percent of income from their own housing units relative to renters. This number is reasonable considering that there are tax benefits for homeownership[16] and that owning a house gives people psychological satisfaction in the form of pride and a sense of safety.

I focus on two sorts of confidence associated with rents: normal and exuberant. Considering the observation that rents in income units were stable over time, the normal rent is taken from the data on nominal rents deflated by nominal income whose average value over the stable period (1975-1998) is 0.196. Rents in the exuberant state are assumed to be 2.5 times higher than normal rents. Among the states of the economy, only the one I call the exuberant state is associated with high rent; the others are associated with normal rent. I argue that rents and prices observed in the data are associated with those on the path towards the exuberant state. The existence of the exuberant state is the key to account for the surge in house prices and stable rents during the boom.

To parameterize the transition probabilities, I assume that the exuberant state is very stable once the economy gets there. For the probability of staying in the exuberant state takes, I assume an extreme value of 0.99. Also, the probability of moving to the boom path from the normal state $(1 - \pi_{N,N})$ is set to be 0.03, which seems to be

[16] One could imagine a situation in which one third of an agent's income goes to mortgage interest payments and the tax rate is 25 percent.

reasonable considering the frequency of booms in asset prices.[17] For example, according to the price-earnings ratio data used in Shiller (2005),[18] there were two rapid booms in stock prices over the sample period of 1881 to 2008. These two booms lasted from 1922 to 1929 and from 1991 to 2000, respectively. Excluding the boom and crash periods, there were about 100 periods.

Somewhat arbitrarily, I assume that there are 12 states. Given the number of states, the structure of the transition probability matrix and the probability of staying in the exuberant state once reached, one can calibrate the probability of a crash for news states by targeting booms' expected durations. I target expected durations to eight periods. Then, the probability of a crash for news states is calibrated as $\pi = 0.268$; this implies that the probability of reaching the exuberant state conditional on the economy in the $(N-1)$th state is $(1-\pi)^{N-2} = 0.044$. Since the probability of going to the boom path is quite small ($1-\pi_{N,N} = 0.03$), the state is an extremely rare event. Table 1 summarizes the parameterization.

| parameter | description | value |
|---|---|---|
| $\beta$ | discount rate | 0.945 |
| $\alpha_H$ | utility from owning a house | 0.0866 |
| $\alpha_R$ | utility from renting a house | 0 |
| $\gamma$ | curvature parameter on matching function | 0.5 |
| $\bar{H}$ | measure of housing units in a location | 1.045 |
| $\bar{N}$ | total measure of households | 1 |
| $\kappa$ | scaling parameter on matching function | 0.144 |
| $w$ | flow income | 1 |
| $m_H$ | moving probability of homeowners | 0.04 |
| $\overline{q}$ | exuberant rent | 0.49 |
| $\underline{q}$ | normal rent | 0.196 |
| $N$ | number of states | 12 |
| $\pi_{1,N}$ | crash probability in the exuberant state | 0.01 |
| $\pi_{N,N}$ | staying probability in the normal state | 0.97 |
| $\pi$ | crash probability in news state | 0.268 |

Table 1: Parameter Values

[17] As long as the probability of moving to the boom path $1-\pi_{N,N}$ is a small number, the parameterization is insignificant for the result.

[18] The most recent data can be found at http://www.econ.yale.edu/~shiller/data.htm

## 5.2 Bounded Bubbles as a Rational Expectations Equilibrium

Assuming that demographics are stable, that is, the measures of renters and homeowners are constant, the price profile is computed as

$$\mathbf{p} = \begin{bmatrix} 9.02 & 7.11 & 6.14 & 5.47 & 5.01 & 4.69 & 4.46 & 4.31 & 4.20 & 4.13 & 4.08 & 3.97 \end{bmatrix}',$$

and house prices substantially increase as the economy moves to higher states. As long as the economy is not in the exuberant state, rent is constant. As I show below, this house price profile satisfies individual rationality. Thus, the recent U.S. housing market can be characterized as a rational expectations sunspot equilibrium.

House prices in the exuberant state are high because of the stability and high rents in that state. To support the constant rent profile for the other states, house prices need to be expected to appreciate moderately. If the house prices are expected to decrease, rents would have to be very high to compensate the expected depreciation. Also, if the expected appreciation is too high, rents would be very small (or even negative) because the value of a vacant house is the discounted sum of future rent payments. Note that the expected value of a house at each news state is characterized by house prices in the state above by one and in the normal state due to the structure of the transition probability matrix. For states with high house values, moving further towards the exuberant state implies a substantial house price increase for moderate expected appreciation; this is because there could be a crash and prices could plummet. By modeling the exuberant state where rents are high, the framework implies that a surge in house prices could occur while rents are stable. This framework generates a boom in house prices with stable rents.

To compare the model prices with the data, I plot prices at the bottom nine states and the recent boom observed from 1998 through 2006. As Figure 9 shows, the model fits fairly well to the empirical surge in house prices. This addresses the main theme of the paper: Due to a steady state indeterminacy in the search and matching framework, self-fulfilling beliefs can drive the economy. If people believe that there is a small chance of reaching the exuberant state, house prices can appreciate substantially while rents remain constant.

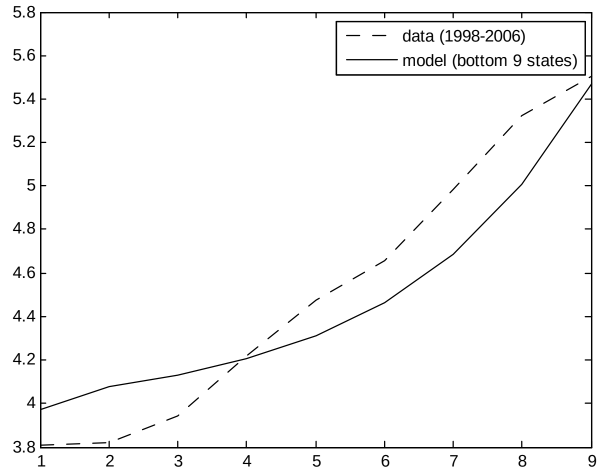


Figure 9: House Prices Derived from the Model

Figure 10 illustrates individual rationality in each state. It indicates that house price bubbles in the model have a bounded feature satisfying the individual rationality condition. Accordingly, the model characterizes a house price bubble as a rational expectations equilibrium. A match surplus in period $t$ is the gain in the values by changing housing tenure $H_t - R_t$ minus the intrinsic value of a housing unit $W_t$. The transaction price $p_t$ should stay inside the set $[W_t, H_t - R_t]$ so that each side of the match appropriates a nonnegative surplus. Even in the exuberant state with very high prices, agents are willing to trade if they match.

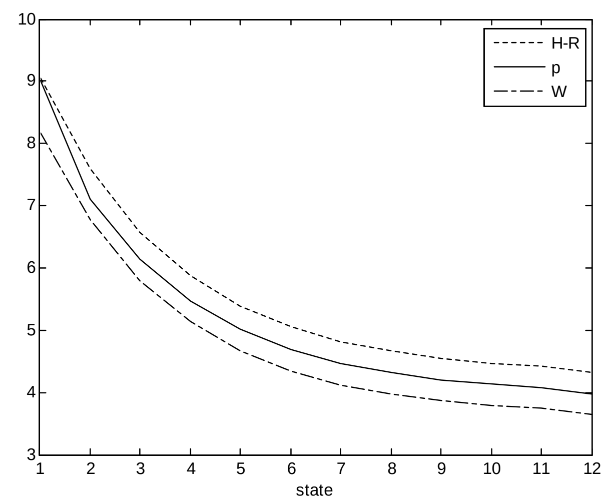


Figure 10: Individual Rationality in Each State

The figure visualizes how the surplus is split between renters and the real estate sector. In the normal state (12th state) the total surplus is equally split.[19] For most states, the surplus is shared approximately equally. Even in the state in which the price appreciates more than 50 percent (4th state), the sharing rate on the renter's side is 0.54. An obvious exception is the exuberant state, where the real estate sector appropriates most surplus. Such an extreme state is still supported as a rational expectations equilibrium. Table 2 summarizes rent, house price and the fraction of surplus that renters appropriate in each state.

| state | rent | house price | sharing rate of renter |
|---|---|---|---|
| 1 (exuberant) | 0.49 | 9.02 | 0.04 |
| 2 | 0.196 | 7.11 | 0.59 |
| 3 | 0.196 | 6.14 | 0.56 |
| 4 | 0.196 | 5.47 | 0.54 |
| 5 | 0.196 | 5.01 | 0.52 |
| 6 | 0.196 | 4.69 | 0.52 |
| 7 | 0.196 | 4.46 | 0.51 |
| 8 | 0.196 | 4.31 | 0.51 |
| 9 | 0.196 | 4.20 | 0.50 |
| 10 | 0.196 | 4.13 | 0.50 |
| 11 | 0.196 | 4.08 | 0.50 |
| 12 (normal) | 0.196 | 3.97 | 0.50 |

Table 2: Prices and Implied Sharing Rate in Each State

Since the model implies a substantial house price appreciation similar to the data, the model can address the probability distribution of house prices. The model is simulated for 30 periods starting from the normal state, which corresponds to the period from 1975 to 2006. This is iterated $10^5$ times. The simulation provides the house price distribution in 2006 with the initial period (1975) in the normal state. Figure 11 presents the result. The vertical axis is the probability and the horizontal axis is the state indexed as in Table 2.

It is highly likely that the economy is in the normal state with probability 0.88. With probability 0.1, the economy is in one of the news states. Since the economy moves

[19] Precisely speaking, the implied sharing rate differs slightly from 0.5 (0.5002) due to randomization. The parameters are calibrated so that the surplus is split equally at the deterministic steady state with the observed rent.

towards the exuberant state one by one and potentially collapses to the normal state from each state, the probability is monotonically decreasing as the economy approaches the exuberant state. Among the news states, the 4th state, which corresponds to the price level observed in 2006, occurs with probability 0.003. The simulation also suggests that the economy is in the exuberant state with probability 0.02. This probability is low but higher than the probabilities of relatively high news states being observed, due to the assumption that the exuberant state is stable once the economy gets there.

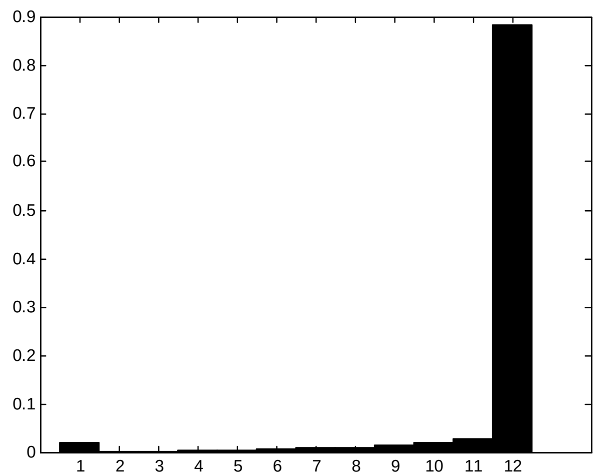


Figure 11: Probability Distribution of House Prices

## 5.3 Sensitivity

This subsection examines the implications of different values of rent and sustainability in the exuberant state. First, I consider variation in rents in the exuberant state, keeping the transition probability matrix. I use rents in the exuberant state of 2.5 times (as used in the analysis above), 2.25 times and two times as much as the normal rent. In Figure 12, I plot the appreciation of house prices as moving from the normal state upward by eight states for different values in rent in the exuberant state. The graph with rent 2.5 times higher than normal corresponds to Figure 9. With rents of 2.25 times and two times as high as the normal rent, house prices appreciate more than 30 percent and about 25 percent, respectively.

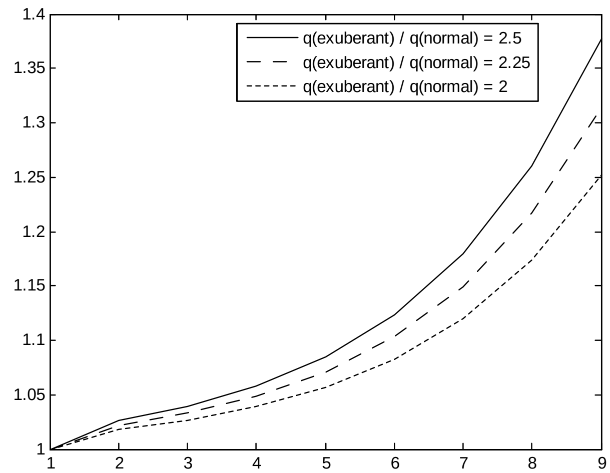


Figure 12: Price Appreciation for Variations in Rents in the Exuberant State

In the second experiment, I consider a case in which the probability of collapsing from the exuberant state to the normal state is the same as the crash probabilities from the news states, $\pi_{1,N} = \pi$. This probability is set as 0.125 by targeting the same expected duration of a boom. The same experiment, which Figure 13 summarizes, shows that there is still a considerable increase in house prices moving towards the exuberant state, though the magnitude is not as large as the case with the original crash probability in the exuberant state. With the selected rent values, prices appreciate more than 15 percent in eight periods.

With the transition probabilities used in the second experiment, exuberant rent 2.5 times higher than normal violates the individual rationality condition. Changes in the transition probability matrix affect not only prices but also households' values. The smaller sustaining probability of the exuberant state lowers prices' upper bound more than it lowers house prices in the exuberant state.

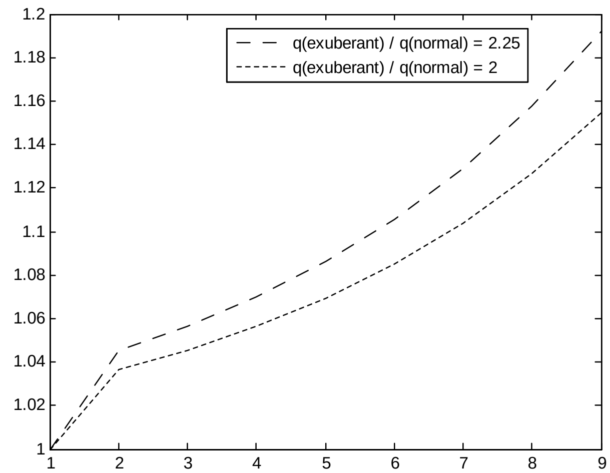


Figure13: Price Appreciation for the Same Crash Probabilities over the States

# 6 Conclusion

This paper focuses on three observations about the U.S. housing market over the recent three decades. First, housing tenure and vacancy were approximately constant. Second, rents were approximately constant. Third, house price data nonetheless showed a large appreciation after the late 1990s. It then provides a model that characterizes those observations as an equilibrium.

The model takes ideas developed in search and matching theory, and is closed with self-fulfilling beliefs. In particular, it focuses on two levels of confidence (normal and exuberant) regarding the housing market that are associated with rents; each defines a deterministic steady state equilibrium.

Sunspots potentially drove the recent boom in the U.S. housing market. This paper formalizes the idea using an equilibrium framework. The model economy evolves according to sunspots due to the construction of the rational expectations equilibrium as randomized over underlying steady state equilibria. Because of multiple underlying deterministic equilibria, a house price bubble driven by self-fulfilling beliefs can exist in a rational expectations equilibrium. The bubble occurs as the economy moves towards the exuberant equilibrium.

# References

[1] Azariadis, Costas (1981) "Self-Fulfilling Prophecies," *Journal of Economic Theory*, pp. 380-396.

[2] Azariadis, Costas and Roger Guesnerie (1986) "Sunspots and Cycles," *Review of Economic Studies*, pp.725-737.

[3] Cass, David and Karl Shell (1983) "Do Sunspots Matter?" *Journal of Political Economy*, pp.193-227.

[4] Davis, Morris, Andreas Lehnert and Robert Martin (2008) "The Rent-Price Ratio for the Aggregate Stock of Owner-Occupied Housing," *Review of Income and Wealth*, pp.279-284.

[5] Farmer, Roger (2009) "Confidence, Crashes and Animal Spirits," mimeo.

[6] Farmer, Roger (2010) *Expectations, Employment, and Prices*, forthcoming, New York: Oxford University Press.

[7] Howitt, Peter and R. Preston McAfee (1987) "Costly Search and Recruiting," *International Economic Review* 28 (1), pp.89-107.

[8] Kiyotaki, Nobuhiro, Alexander Michaelides and Kalin Nikolov (2008) "Winners and Losers in Housing Markets," mimeo.

[9] Krainer, John (2001) "A Theory of Liquidity in Residential Real Estate Markets," *Journal of Urban Economics*, pp.32-53.

[10] Mortensen, Dale and Christopher Pissarides (1994) "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, pp.397-415.

[11] Ngai, Rachel and Silvana Tenreyro (2009) "Hot and Cold Seasons in the Housing Market," mimeo.

[12] Peterson, Brian (2009) "Fooled by Search: Housing Prices, Turnover and Bubbles," mimeo.

[13] Piazzesi, Monika and Martin Schneider (2009) "Momentum Traders in the Housing Market: Survey Evidence and a Search Model," *American Economic Review: Papers and Proceedings*, pp.406-411.

[14] Rubinstein, Ariel and Asher Wolinsky (1987) "Middlemen," *Quarterly Journal of Economics*, pp.581-593.

[15] Shiller, Robert (2005) *Irrational Exuberance*, 2nd ed., Princeton, NJ: Priceton University Press.

[16] Shiller, Robert (2007) "Understanding Recent Trends in House Prices and Home Ownership," mimeo.

[17] Shimer, Robert (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, pp.25-49.

[18] Van Nieuwerburgh, Stijn and Pierre-Olivier Weill (2009) "Why Has House Price Dispersion Gone Up?" mimeo.

[19] Weil, Philippe (1987) "Confidence and the Real Value of Money in an Overlapping Generations Economy," *Quarterly Journal of Economics*,. pp.1-22.

[20] Wheaton, William (1990) "Vacancy, Search, and Prices in a Housing Market Matching Model," *Journal of Political Economy*, pp.1270-1292.

[21] Williams, Joseph (1995) "Pricing Real Assets with Costly Search," *Review of Financial Studies*, pp.55-90.