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Bank lending to the Private Sector and GDP growth: Thresholds and Returns

Demetris Koursaros*, Nektarios A. Michail** and Christos S. Savva***

Abstract

We examine the relationship between lending to the private sector and GDP growth using a two-period model and test model conclusions through a Smooth Transition Conditional Correlation (STCC) model for the G7 countries. Theory suggests that the correlation between private lending and growth is positive and this relationship exhibits diminishing returns after a threshold. The empirical exercise confirms that this relationship holds, and while thresholds exist for most countries, the correlation between private lending and growth is never negative. Overall, the evidence indicates that policy should not emphasise the level of lending but its allocation in the economy.

Keywords: private debt, correlation, bank lending, threshold, policy

JEL Classification: E51, E60, C32

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Bank lending to the Private Sector and GDP Growth: Thresholds and Returns

1. Introduction

Is bank lending always beneficial to the economy or can too much finance actually harm growth? The question, addresses both intellectual as well as policy issues. For example, the euro area scoreboard indicators consider a country to be over-indebted if it exceeds the 130% private debt to GDP ratio. However, the issue of whether a ratio of 100% is actually better (or worse) than a 140% ratio and how much is growth affected by an increase in the stock of loans has yet to be clarified.

In this paper, we fill this gap by examining the effects of private bank lending in the economy through a two-period model. Theory suggests that even when changes in lending are due to monetary easing, a positive relationship between finance and growth exists, while this relationship exhibits diminishing returns after a certain threshold. This threshold can be viewed as the point where new projects are either not as promising as those already funded or they lack collateral to attract a loan. Subsequently, more funds are channeled to existing projects, thus lowering returns.

Furthermore, the theoretical conclusions are tested by employing a smooth transition conditional correlation (STCC) model. The empirical evidence also suggests that the finance-growth relationship is always positive but exhibits diminishing returns after country-specific thresholds. Both the theoretical and the empirical exercises conclude that there is a positive correlation between finance and growth, with the relationship exhibiting diminishing returns. Overall, the results suggest that, other things being constant, private lending promotes GDP growth, at all levels. As such, macro-prudential policies should not emphasise how much lending exists in an economy, but how the allocation of these loans affects the workings of the country.

This work empirically supports some of the most important theoretical contributions in the literature such as Eggertson and Krugman (2012) who suggest what really matters is the distribution of debt and not its level. As the rest of the paper demonstrates, the level of debt makes a difference only in the size of its relationship to the real economy but it still retains the same (positive) effect. Thus, it can be inferred that if the stock of loans does not affect growth all that is left is that the distribution of this stock matters. This conclusion holds important policy implications, in that macro-prudential regulation should emphasise how loans are distributed in the economy and not simply focus on their level.

Other empirical studies in this area have, over time, come out with conflicting results. One of the first studies was that of King and Levine (1993) who show that higher rates of financial development are positively correlated with higher growth in the short-run, while the predetermined component of financial development is a good predictor of long-run growth over a 10 to 30 year horizon. In addition, they note that higher rates of financial development are strongly associated with future rates of capital accumulation and improve capital efficiency. Complementing their results, DeGregorio and Guidotti (1995) find that the proxy of bank credit to GDP is positively correlated with growth. The authors use a large cross-country sample and attribute the negative correlation in a Latin America panel to financial liberalisation in a poor regulatory environment.

In contrast to these, Demetriades and Hussein (1996), and Arestis and Demetriades (1997), show that the causality of the relationship between financial development and economic growth is highly country-specific. In addition, they provide little support that finance is a leading sector, even though they find evidence of bi-directionality and reverse causation. The bi-directionality of the relationship was further examined in Shan et al (2001) who also find little evidence that finance leads growth using Granger causality tools. Diverging from these results, Calderón and Liu (2003) find that financial development generally leads to economic growth while the longer the sampling interval, the larger the effect of financial development on economic growth.

Arestis et al (2001), using data from five developed economies, find that even though both banks and stock markets are able to promote financial development, the former have a much stronger effect and the latter's contribution may have been exaggerated in cross-country studies. In addition, Demetriades and Luintel (1996) suggest that financial deepening may also have an influence on economic growth. These results are supported by other studies, such as Odedokun (1996), who demonstrates that financial intermediation promotes growth in 85% of the sample countries employed, and Hassan et al (2011) who find a positive correlation between finance and growth in developing countries. A relatively recent review of the literature by Levine (2005) suggests that both financial intermediaries and markets matter for growth and that reverse causality alone does not drive this relationship.

Policy issues, on the other hand, have put forth the question of whether a limit exists after which financial growth is just "too much" and can harm economic development. To address this question, Cechetti et al (2011) use a panel of 18 OECD countries from 1980 until 2010 and find that the thresholds for household and corporate debt lie at 85% and 90%, respectively. Similar threshold levels (of less than 100% of GDP) are further supported in Cechetti and Kharroubi (2012) as well as Law and Singh (2014) using a similar methodology.

However, this result appears rather counter-factual: the majority of developed countries record private debt to GDP ratios higher than this level while still registering positive growth rates. In addition, the use of just one methodology to gauge a result which highly affects policy is by no means enough to justify a conclusion. In the Sections which follow, we present, in contrast to the authors mentioned in the previous paragraph, a two-period model and results from an empirical specification which prove that debt does not harm growth at any given private sector debt to GDP ratio.

2. Theory: a two-period model

Suppose there is an island which lives for two periods. There are households which are both savers and farmers. Some households produce in the first period and thus enjoy income from farming while the rest can produce in period two. The ones that produce in the first period deposit some of this income in a bank, to transfer part of their wealth to the following period where they will have no other sources of income. These deposits are transferred by the bank to the households that are capable of producing in the current period, in the form of loans. Bank lending in the initial period comes from bank reserves. The farmers simply plant crops, relying on bank loans and on the tools each one possesses, which can also be used as collateral for the loans. The households do not bail out insolvent farmers and thus collateral goes to the bank upon default.

2.1. Households

In the first period, the households that are both producers and savers decide upon how much to consume and how much to save, i.e. transfer to the second period through bank deposits which pay a rate of R_t^d for the period. Initial wealth comes from deposits along with interest payments, income from farming and profit from banks. Thus, all income in each period goes to the households which are currently producing. The producing households maximise utility for two periods

$$\max U(x_t) + \beta E_t U(x_{t+1}) \tag{1}$$

subject to the constraints

$$x_t + D_t = \Pi_t \tag{2}$$

where x_t denotes consumption and D_t denotes deposits at the bank. As stated before, initial wealth comes from current production (Π_t). In the following period, consumption can be at most what has been saved in the previous period when income was available, times gross interest on deposits R_t^d :

$$x_{t+1} = R_t^d D_t \tag{3}$$

In the second period, all income is accumulated by households which produce, creating an incentive for some households to save and an incentive for other households to take loans. The first order condition for consumption and deposits in the two periods produces the following Euler equation:

$$U'(x_t) + \beta E_t U'(x_{t+1}) R_t^d \tag{4}$$

Assuming that the utility function is

$$U(x_t) = \frac{x^{1-\sigma}}{1-\sigma} \tag{5}$$

Substituting the utility function (5) in the Euler equation (4) yields

$$\frac{1}{(\Pi_t - D_t)^\sigma} = \beta \frac{1}{(R_t^d D_t)^\sigma} R_t^d \tag{6}$$

If we assume for simplicity, and without any loss of generality, that $\sigma = \frac{1}{2}$ then we get

$$R_t^d = \frac{1}{\beta^2}\frac{D_t}{\Pi_t - D_t} \tag{7}$$

2.2. Farmers

The farmers (i.e. the households which produce in the current period) borrow from the banks an amount L_{it} of the real good which is used to produce $F(L_{it})$ units of the real good in the following period. The production function exhibits diminishing returns to scale

$$F(L_{it}) = z_{it} A L^{a}{}_{it} \tag{8}$$

where A is a productivity parameter common to all farmers, the value of which is known to everyone. The idiosyncratic productivity parameter, z_{it}, is a draw from a known distribution. The only uncertainty in the model comes from this parameter. Households (and banks) are entitled to a continuum of such assets and thus *ex ante* and *ex post* returns are entirely the same.

Households which seek loans are by assumption the ones which are entitled to farmers' profits. Farmers eligible for bank loans need to provide some form of collateral. Given collateral values and the lending rate, the farmers maximise the expected profit by choosing the loan amount they seek:

$$E_t \Pi_{t+1} = \max_{L_{it}} \int_{\hat{z}_t}^{\infty} \left[F(L_{it}) - R_t^L L_{it} \right] dF(z) - \int_{-\infty}^{\hat{z}_t} C_{t+1} dF(z) \tag{9}$$

If the productivity value drawn is high enough and the no-default state is realised, the farmer produces $F(L_{it})$ and repays the whole amount back. In the default state, the farmer can expect to lose no more than his own collateral C_{t+1}, no matter what the draw from the productivity distribution might have been. The default state is defined through the productivity reservation point

$\hat{z}_t$, where the lender starts to take part on the loss. The reservation productivity point is where the loss from the production ($z_{it} A L^a{}_{it} - R_t^L L_{it}$) equals $-C_{t+1}$. Solving this for $\hat{z}_t$ implies that

$$\hat{z}_t = \frac{R_t^L L_{it} - C_{t+1}}{A L^a{}_{it}} \tag{10}$$

If we define the probability of no default as p_{it} and the truncated mean of productivity given that we are in the no default zone (i.e. $E_t(z_{it} \mid z_{it} < \hat{z}_t)$) as $\bar{z}_t^e$ then the expected profit in equation (9) becomes

$$\max\ p_{it} \bar{z}_t^e A L^a{}_{it} - p_{it} R_t^L L_{it} - (1 - p_{it}) C_{t+1} \tag{11}$$

Supposing that z_{it} is drawn from the uniform distribution $U(0,1)$, this implies that

$$\bar{z}_t^e \equiv E\left(z_{it} \middle| z_{it} < \hat{z}_t \right) = \frac{1}{2}(1 + \hat{z}_t) \tag{12}$$

In a similar manner, the expected productivity in the default state ($\bar{z}_t^d$) is:

$$\bar{z}_t^d \equiv E\left(z_{it} \middle| z_{it} < \hat{z}_t \right) = \frac{1}{2}\hat{z}_t \tag{13}$$

Another implication of the uniform distribution is that the productivity of default is linear in the reservation productivity $\hat{z}_t$. That is

$$p_{it} = 1 - \hat{z}_t \tag{14}$$

Using these, the first order condition of the farmers' maximization problem (equation 11) is

$$a \bar{z}_t^e A L_{it}^{a-1} = R_t^L \tag{15}$$

Substituting (10) in (15), the reservation productivity which is also the probability of default under a uniform distribution becomes

$$\hat{z}_t = a\bar{z}_t^e A - \frac{E_t C_{t+1}}{L_{it}^a} \tag{16}$$

2.3. Banks

The banks make one period loans and get deposits of the same maturity. Initially, bank capital V_t is exactly equal to bank reserves. Banks constitute the only source of financing to the private sector and thus lending defines the output of the economy as firms need loans to be able to produce. Output, Y_t is related to loans L_{it} to each of the N_t farmers (firms) created last period as follows:

$$Y_t = N_{t-1} F\left(L_{it-1}\right) \tag{17}$$

The banks start the period with bank capital V_t, which as stated in the previous paragraph, also represents the reserves in the banking system[1]. The central bank can also boost lending in the economy by injecting fresh money in banks at the beginning of the period. Increases in V_t by the central bank thus increase the lending capacity of the bank. If the bank can adjust both the number of loans and the amount lent to each firm then increases in liquidity in banks are going to increase income in the economy. However, lending is concentrated to those that possess valid collateral and even though bank reserves can freely expand, the lending opportunities will eventually become exhausted. Inequality in the distribution of assets that can be used as collateral is important for the point beyond which new loans cannot be made. We are interested in investigating what happens if

[1] Since contracts are fully repaid and last for a period, the bank capital in the end of the period is actually the bank reserves in the bank's balance sheet, for the asset side to equal the liability side.

bank liquidity increases while the number of new loans, N_t, is fixed. We assume the number of loans is controlled by the bank headquarters while the amount available for lending to each farmer is controlled by a loan officer. The aggregate number of loans, Γ_t, depends on deposits and capital according to

$$\Gamma_t = D_t + V_t \tag{18}$$

As stated before, in the event of default, the bank gets the collateral. The objective of the loan officer is to maximise profits from granting a single loan, i.e.

$$\max_{L_{it}} p_{it} R_t^L L_{it} + (1 - p_{it}) \bar{z}_t^d A L_{it}^a + (1 - p_{it}) E_t C_{it+1} - L_{it} R_t^d \tag{19}$$

This implies that in a no default state which happens with probability p_{it}, the bank gets $R_t^L L_{it}$ and in the default state it gets all the firm's revenue which, in expectation, equals to $\bar{z}_t^d A L_{it}^{\alpha}$, where $\bar{z}_t^d$ is as stated in equation (13), and the bank also gets the collateral of the firm that is expected to be worth $E_t C_{it+1}$. Each loan officer, receives from headquarters an amount L_{it} at cost R_t^d. The first order condition which determines loan supply is

$$p_{it} R_t^L + a(1 - p_{it}) A \bar{z}_t^d L_{it}^{a-1} = R_t^d \tag{20}$$

Rearranging it we get

$$L_{it} = \left[\frac{a(1 - p_{it}) A \bar{z}_t^d}{R_t^d - p_{it} R_t^L} \right]^{\frac{1}{1-a}} \tag{21}$$

2.4. Equilibrium in the Market for Loans

To solve the loan supply and loan demand problem, we substitute equation (15) into (21) to obtain

$$L_{it} = \left[\frac{a(1-p_{it})A\overline{z}_t^d}{R_t^d - p_{it}\overline{z}_t^e AL_{it}^{a-1}} \right]^{\frac{1}{1-a}}$$

and substituting the definitions of $\overline{z}_t^e$ and $\overline{z}_t^d$ from equations (12) and (13) we get

$$L_{it} = \left[\frac{a(1-p_{it})A\frac{1}{2}\hat{z}_t}{R_t^d - p_{it}\frac{1}{2}(1+\hat{z}_t)aAL_{it}^{a-1}} \right]^{\frac{1}{1-a}}$$

If we also substitute (14) in the above we get that

$$L_{it} = \left[\frac{a(\hat{z}_t)A\frac{1}{2}\hat{z}_t}{R_t^d - (1-\hat{z}_t)\frac{1}{2}(1+\hat{z}_t)aAL_{it}^{a-1}} \right]^{\frac{1}{1-a}} \Rightarrow L_{it}^{1-a}R_t^d - L_{it}^{1-a}(1-\hat{z}_t)\frac{1}{2}(1+\hat{z}_t)aAL_{it}^{a-1} = a(\hat{z}_t)A\frac{1}{2}\hat{z}_t$$

which simplifies to $L_{it}^{1-a}R_t^d - \frac{1}{2}(1-\hat{z}_t^2)aA = aA\frac{1}{2}\hat{z}_t^2$ and further to $L_{it}^{1-a}R_t^d - \frac{1}{2}(1-\hat{z}_t^2)aA = aA\frac{1}{2}\hat{z}_t^2$

and to $L_{it}^{1-a}R_t^d = \frac{1}{2}aA$, finally yielding that

$$L_{it} = \left[\frac{1}{2}\frac{aA}{R_t^d} \right]^{\frac{1}{1-a}} \tag{22}$$

As the above suggests, the amount of loans to each agent is a decreasing function of the deposit rate. The equilibrium lending rate (15) after substituting (22) becomes:

$$a\bar{z}_t^{e} A\left[\frac{1}{2}\frac{aA}{R_t^d}\right]^{\frac{a-1}{1-a}} = R_t^L$$

which simplifies to $a\bar{z}_t^{e} A\left[2\frac{R_t^d}{aA}\right] = R_t^L$ and further to $2R_t^d \bar{z}_t^e = R_t^L$, which after using the definition of $\bar{z}_t^e$ in (12) reduces to:

$$R_t^L = (1+\hat{z}_t)R_t^d \tag{23}$$

suggesting that the lending rate is simply a markup over the deposit rate. This markup is increasing in the probability of default.

Since the collateral cannot be of arbitrary value, the loan collateral is a percentage of the lending amount and has to follow the evolution of the amount due, i.e.

$$E_t C_{t+1} = (1-\phi)R_t^L L_{it} \tag{24}$$

where $(1-\phi)$ can be viewed as the loan to value ratio. To get the value of the probability of default, we substitute (23) and (25) into (10)

$$\hat{z}_t = \frac{(1+\hat{z}_t)R_t^d L_{it} - (1-\phi)(1+\hat{z}_t)R_t^d L_{it}}{AL^a{}_{it}}$$

which simplifies into $\hat{z}_t = \frac{(1+\hat{z}_t)R_t^d L_{it} - (1-\phi)(1+\hat{z}_t)R_t^d L_{it}}{AL^a{}_{it}} = \frac{\phi(1+\hat{z}_t)R_t^d L_{it}}{AL^a{}_{it}} = \frac{\phi(1+\hat{z}_t)R_t^d L_{it}^{1-a}}{A}$

Substituting (22) into the above we get $\hat{z}_t = \frac{\phi(1+\hat{z}_t)R_t^d\left[\frac{1}{2}\frac{aA}{R_t^d}\right]^{\frac{1-a}{1-a}}}{A}$ and simplifying yields

$$\hat{z}_t = \frac{1}{2} a\phi(1+\hat{z}_t)$$

Then solving for $\hat{z}_t$, the probability of default becomes

$$\hat{z}_t = \frac{\phi\alpha}{2-\phi\alpha} \tag{25}$$

i.e. the probability of default is a function of the collateral required, and the production function parameter. Thus, the probability of default becomes a constant at equilibrium.

2.5. The effects of an increase in lending

Imagine that the central bank is printing money to boost initial reserves in the banking system. This monetary easing can be viewed as a change in policy and not in the fundamentals of the economy, in which case an increase in lending would obviously be growth-enhancing. The model is a two-period model and thus inflation is by assumption fixed as this is the short run. If the bank can grant more loans that are not subject to diminishing returns and also adjust the amount to be granted to each individual then the question is trivial. More lending is going to promote higher growth. To make the exercise both more challenging and more realistic, too much lending eventually is going to deplete new lending opportunities (or simply loan seekers would lack valid collateral) and thus N_t is going to become fixed. In a realistic situation, an increase in the ability of the bank to grant loans (in this case, an increase in bank capital or reserves) does not mean that it will offer a loan to every person in the economy but it would rather choose to ration credit and thus be bound by those persons which have the ability to borrow, i.e. possess collateral. Plugging (18) into (7) and we get:

$$R_t^d = \frac{1}{\beta^2}\frac{\Gamma_t - V_t}{\Pi_t - \Gamma_t - V_t}$$

and using the fact that $\Gamma_t = N_t L_{it}$, i.e. that the aggregate number of loans equals the amount lent to each agent multiplied by the number of agents, and that $\Pi_t = sY_t$, i.e. that household wealth is a fraction of output, yields

$$R_t^d = \frac{1}{\beta^2}\frac{N_t L_{it} - V_t}{sY_t - N_t L_{it} + V_t} \tag{26}$$

and differentiating it with respect to bank capital V_t yields (after simplifications)[2],

$$\frac{dR_t^d}{dV_t} = \frac{-sY_t}{\beta^2\left(sY_t - N_t L_{it} + V_t\right)^2 + \frac{1}{1-a}\left[\frac{aA}{2}\right]^{\frac{1}{1-a}} N_t \left[R_t^d\right]^{\frac{-1}{1-a}-1} sY_t} < 0 \tag{27}$$

Through the chain rule, it is known that $\frac{dY_{t+1}}{dV_t} = aN_t A L_{it}^{a-1}\frac{dL_{it}}{dR_t^d}\frac{dR_t^d}{dV_t}$ and thus the derivative of output with respect to bank capital is positive,

$$\frac{dY_{t+1}}{dV_t} = 2\frac{1}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}} \frac{sY_t N_t}{\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}} + \frac{1}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}} sY_t N_t \left(R_t^d\right)^{-1}} > 0 \tag{28}$$

and the second derivative of output with respect to bank capital after simplifying with (7) and the derivative of deposits with respect to bank capital $\left(\frac{dD_t}{dV_t}\right)$ becomes:

[2] Full mathematical workings for the calculation of the first derivative can be found in Supplement 1 accompanying this paper, while workings for the calculation of the second derivative can be found in Supplement 2.

$$\frac{d^2Y_{t+1}}{dV_t^2} = \frac{\frac{2sY_tN_t}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-4-a}{1-a}}\left[\frac{1+4a}{1-a}\left(D_t\right)^2 + \frac{sY_tN_t}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{6-a}{1-a}} + 2\frac{\left(D_t\right)^3}{sY_t}\right]\frac{dR_t^d}{dV_t}}{\left[\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}} + \frac{1}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}} sY_tN_t\left(R_t^d\right)^{-1}\right]^2} \tag{29}$$

which is negative since $\frac{dR_t^d}{dV_t} < 0$.

Overall, the effects of an increase in lending, unrelated to the fundamentals of the economy, can be summarised as follows: the first derivative suggests that the effect of an increase in bank lending on growth (measured through an increase in bank capital since lending is an endogenous variable) is positive. However, this effect has diminishing returns as the second derivative suggests. By and large, the model indicates that there is no point after which private bank lending is harmful to the economy, but the effect of each subsequent loan differs, since diminishing returns are expected after some point. In the section which follows, we present an empirical investigation of these results using data from the G7 countries through a Smooth Transition Conditional Correlation (STCC) model.

3. Data and Empirical Methodology

3.1. Data

To empirically examine the results of the previous section, we use nominal GDP and nominal lending in the economy for the G7 countries to construct the private debt to GDP ratio, and the GDP growth rate to examine the relationship between finance and growth. Data series for total credit in the economy were obtained from Dembiermont et al (2013) at BIS, who construct a long series on total and domestic bank credit to the private non-financial sector.

We choose to employ total credit to the economy to capture the effects in local credit ratios instead of that from just domestic banks since the latter may not be representative. More specifically, in the sample of developed economies there is a large deviation between the two. Additional support for the selection of total credit in the economy is that in common policy evaluations where credit is considered, such as the EU Scoreboard Indicators, our selection better characterises the policy evaluation. [3,4]

3.2. Econometric Methodology

In this section we introduce the bi-variate GARCH model with Smooth Transition Conditional Correlation (STCC), proposed by Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005). This model enables us to test the time-varying relationship between finance and growth.

Consider a time series of two variables (in this case the private debt to GDP and economic growth) is $\{y_t\}$, $t = 1, \ldots, n$, $y_t = (y_{1,t}, y_{2,t})'$, the stochastic properties of which are assumed to be described by the following model

$$y_t = c_0 + \sum_{k=1}^{p} \phi^k y_{t-k} + \varepsilon_t \tag{30}$$

[3] For robustness purposes, we have also tested for thresholds by employing the amount of credit supplied by domestic banks as well as real GDP and have found very similar responses. Results are available upon request.
[4]http://ec.europa.eu/economy_finance/economic_governance/macroeconomic_imbalance_procedure/mip_scoreboard/index_en.htm

where $\phi^k = \begin{bmatrix} \phi_{11}^k & \phi_{12}^k \\ \phi_{21}^k & \phi_{22}^k \end{bmatrix}$, $k = 1,\ldots,p$ captures any possible own past effects and any past effects from one variable to the other. The error process of (33) is assumed to be time varying

$$\varepsilon_t \mid I_{t-1} \sim N(0, \Sigma_t), \tag{31}$$

where I_{t-1} is the information set consisting of all relevant information up to and including time $t-1$, and N denotes the bivariate normal distribution. The conditional covariance matrix of ε_t, Σ_t, is assumed to follow a time-varying structure given by

$$\Sigma_t = E[\varepsilon_t \varepsilon_t' \mid I_{t-1}], \tag{32}$$

$$\sigma_{11,t} = \omega_1 + \alpha_1 \varepsilon_{1,t-1}^2 + \beta_1 \sigma_{11,t-1}, \tag{33}$$

$$\sigma_{22,t} = \omega_2 + \alpha_2 \varepsilon_{2,t-1}^2 + \beta_2 \sigma_{22,t-1}, \tag{34}$$

$$\sigma_{12,t} = \rho_t(\sigma_{11,t}\sigma_{22,t})^{1/2} \tag{35}$$

$$\rho_t = \rho_0(1-G(s_t;\gamma,c)) + \rho_1 G(s_t;\gamma,c) \tag{36}$$

where it is assumed that the conditional variances $\sigma_{11,t}$ and $\sigma_{22,t}$ both follow a GARCH(1,1) specification.

To capture temporal changes in the contemporaneous conditional correlation ρ_t the logistic function is employed, in line with Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005).

$$G(s_t;\gamma,c) = \frac{1}{1+\exp(-\gamma(s_t - c))}, \qquad \gamma > 0, \tag{37}$$

where s_t is the transition variable, and γ and c determine the smoothness and location, respectively, of the transition between the two correlation regimes.[5] The starting values of γ and c are determined by a grid search while the transition variable is initially (for a sample size n) described as a function of the debt to GDP ratio, with the likelihood function maximised in one step.

The resulting STCC-GARCH model is able to capture a wide variety of patterns of change. ρ_0 and ρ_1 represent the two extreme states of correlations between which the conditional correlations can vary over time according to the transition variable s_t. Differing ρ_0 and ρ_1 imply that the correlations increase ($\rho_0 < \rho_1$) or decrease ($\rho_0 > \rho_1$), with the pace of change determined by the slope parameter γ. This change is abrupt for large γ, and becomes a step function as $\gamma \rightarrow \infty$, with more gradual change represented by smaller values of this parameter (in the estimation, the maximum value of the γ parameter is set to be 100). Parameter c defines the location of the transition c indicates the mid-point of any change in the correlation due to a change in the private debt to GDP ratio. When the transition variable has values less (greater) than c, the correlations are closer to the state defined by ρ_0 (ρ_1).

To test whether the STCC specification is an adequate one, a Lagrange Multiplier (LM) test is employed for the validity of this model against a constant conditional correlation model (CCC) as per Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2009). The results of the test show

[5] The transition function $G(s_t;\, \gamma,\, c)$ is bounded between zero and one, so that, provided there are valid correlations lying between -1 and +1, the conditional correlation ρ_t will also lie between −1 and +1.

that the constant correlation null hypothesis is rejected in all countries except Japan and thus STCC models are estimated for the rest of the G7 countries.

Subsequently, we examine whether a second change is needed, again using an LM test as suggested by Silvennoinen and Terasvirta (2007). The estimation finds no evidence of a need for a double smooth transition conditional correlation (DSTCC), thus we do not proceed with the estimation of this model.[6]

4. Empirical Results

Table 1 presents the descriptive statistics for the G7 countries in the sample. Overall, all sample correlations in the G7 are greater than 0.35, with the largest value recorded in France (0.64) followed by Japan (0.56). As expected, the finance-growth relationship does not exhibit a one-to-one correlation however, it is positive across the sample countries. Thus, the next step is the estimation of the threshold values after which the correlation between finance and growth switches.

Table 1 – Descriptive Statistics

Country Name	**Sample Correlations**	**Sample Size**	**Source**
Canada	0.416	1970Q1-2013Q4	BIS and globalfinancialdata.com
France	0.644	1969Q4-2013Q4	BIS and globalfinancialdata.com
Germany	0.552	1961Q1-2013Q4	BIS and globalfinancialdata.com
Italy	0.363	1989Q4-2013Q4	BIS and globalfinancialdata.com
Japan	0.561	1981Q1-2013Q4	BIS and globalfinancialdata.com
United Kingdom	0.354	1963Q1-2013Q4	BIS and globalfinancialdata.com
United States	0.412	1952Q1-2013Q4	BIS and globalfinancialdata.com

The results of the empirical estimation for the G7 can be found in Table 2. As is evident from the table, all countries exhibit a similar pattern of responses to the private debt to GDP ratio, with

[6] Constant correlation and DSTCC test results can be presented upon request.

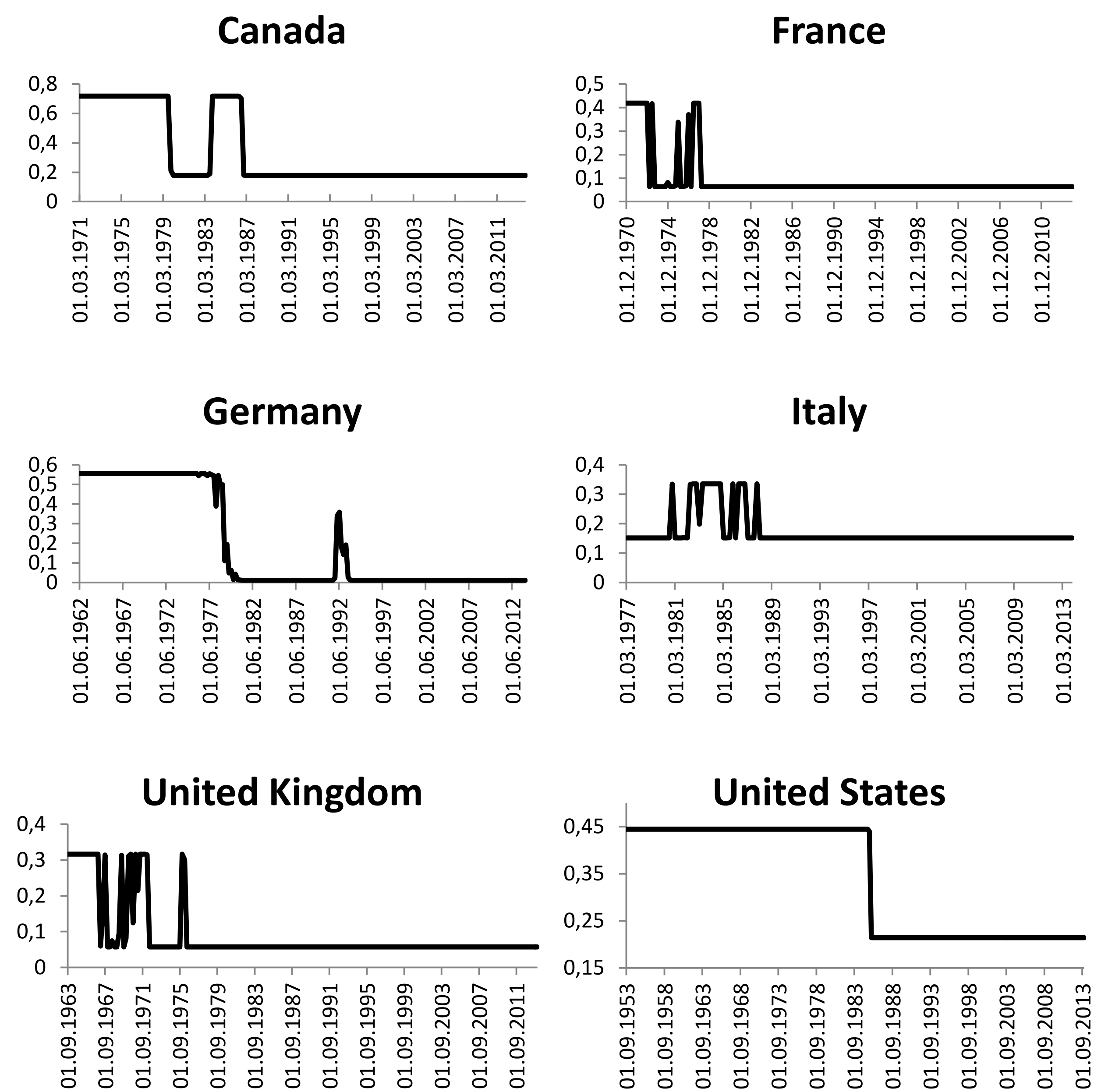
correlations being higher before the threshold and reducing after it. This result confirms the findings in Section 2.5, suggesting that even increases in lending attributed to monetary easing and not changes in the fundamentals of the economy can be growth-enhancing. The size of this correlation is, however, highly country-specific and ranges from almost 98% change in the correlation in Germany to approximately 52% in Italy.

Table 2 – STCC Results

	Canada	**France**	**Germany**	**Italy**	**Japan (CCC)**	**United Kingdom**	**United States**
Correlation before the threshold (p0)	0.712	0.419	0.557	0.335	0.024	0.316	0.444
Correlation after the threshold (p1)	0.179	0.064	0.011	0.152	N/A	0.057	0.214
Threshold value (% of GDP)	85.3%	65.9%	106.6%	50%	N/A	38.7%	105.4%
Smoothness of transition (γ)	100	100	43.49	100	N/A	100	100
Double threshold test	rejected at 5%	rejected at 5%	rejected at 5%	rejected at 5%	N/A	rejected at 5%	rejected at 5%
%change in Correlation	74.8%	84.7%	97.9%	54.6%	N/A	81.9%	51.9%

It can also be observed that the abruptness in the change of these correlations is not related to the size of the change. In Germany, for example, where the largest change in correlation is recorded, the abruptness is approximately half of that in the other countries, as Figure 1 also indicates. The abruptness in the change is blunter in the cases of Canada, France, Italy, the UK and the US, where there appears to be a regime switch.

Figure 1 – Transition of Correlations (Y-axis reflects correlation value)



With regards to the point of change (i.e. the threshold values) the change in correlation for most countries occurs after relatively low values (for France, Italy and the UK the threshold is lower than 66% of GDP), whereas for Canada the value stands at approximately 85%. For the remaining two countries the threshold value appears at higher levels of the ratio such as 105% for the US and

106% for Germany. The idiosyncratic nature of the results underlines the fact that threshold values and correlations can differ significantly across countries and an effort to pin-point a value which would be suitable for all might result in numbers which can potentially under- or over-estimate the true threshold.

The link between the results of this Sections and the ones of Section 2 is straightforward. The threshold level can be viewed as the point where either new projects are not as promising (high-return) as those already funded or the entrepreneurs promoting them lack collateral. As such, more funds are channeled to already existing projects, causing diminishing returns to get underway.

5. Policy Implications

The common element which arises from this analysis is that the correlation between private lending and growth is always positive under every private debt to GDP value. Theory suggests that after a threshold value, diminishing returns kick in, while the empirical results confirm that the correlation exhibits diminishing returns in 6 out of 7 countries (and is constant in the other), slowing its pace after the threshold value, but never switches signs. However, stress should not be placed on the specific values of the thresholds nor on values of the estimated correlations as these may change (number-wise) with the choice of sample. Instead, emphasis should be placed on the broader picture which, in accordance to the two-period model of Section 2, suggests that the level of lending *per se* does not harm growth at any threshold, but instead shows diminishing returns to scale.

These points justify important policy implications: as the results indicate, if the level of the stock of loans does not matter thus, then the only thing which remains is the distribution. As such, macro-prudential policies should not emphasise how much lending exists in an economy, but how the allocation of these loans affects the workings of the country. For example, diminishing returns can take effect not because new projects have lower returns but because the entrepreneurs backing them do not possess collateral to attract a loan.

Consequently, policy should aim at relaxing credit constraints faced by entrepreneurs with promising projects and avoid over-lending to specific sectors of the economy. As already discussed, the latter can occur when new projects lack collateral to attract loans and thus banks lend in already existing projects. In addition, monetary easing when profitable opportunities by entrepreneurs with collateral are limited can be ineffective since it will be channeled to existing projects that are considered less risky for banks (collateral-wise).

6. Conclusions

This study suggests that there is a positive relationship between finance and growth (even in the case of a simple monetary easing), with this relationship exhibiting diminishing returns, after a certain threshold. This threshold can be viewed as the point where new projects are either not as promising as those already funded or they lack collateral to attract a loan. Thus, more funds are channeled to already existing projects lowering returns. Empirical evidence also suggests that, in accordance with the theory, there is no point or threshold after which the amount of loans is harmful to economic growth. In addition, the empirical exercise confirms that thresholds exist, but the relationship between bank lending and growth remains positive, albeit exhibiting diminishing returns. The results also suggest that the determination of a threshold value is highly country-specific and does not affect the size of the change in correlation. This stresses the fact that policies aimed at safe-guarding the economy from potential pitfalls should not aim at retaining the level of lending under some specific threshold but instead aspire to monitor the allocation of this debt in the economy in order to prevent over-heating of specific sectors by lending to already existing projects instead of new ones. For the same reason, monetary easing can be ineffective when profitable opportunities by entrepreneurs with collateral are limited since banks will channel it to existing projects that are considered less risky, with respect to collateral.

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Supplement 1

Mathematical workings for the calculation of the first derivative of output with respect to a change in lending

From equation (22) in the main text, we have that $L_{it} = \left[\frac{1}{2}\frac{aA}{R_t^d}\right]^{\frac{1}{1-a}}$ or $L_{it} = \left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}}$

Differentiating with respect to R_t^d yields: $\frac{dL_{it}}{dR_t^d} = \frac{-1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}$ (1)

Using equation (26) in the main text we know that $R_t^d = \frac{1}{\beta^2}\frac{N_t L_{it} - V_t}{sY_t - N_t L_{it} + V_t}$ which we can differentiate with respect to V_t:

$$\frac{dR_t^d}{dV_t} = \frac{1}{\beta^2}\frac{\left(N_t\frac{dL_{it}}{dV_t} - 1\right)\left(sY_t - N_t L_{it} + V_t\right) - \left(N_t\frac{dL_{it}}{dV_t} + 1\right)\left(N_t L_{it} - V_t\right)}{\left(sY_t - N_t L_{it} + V_t\right)^2}$$

Or after simplifying:

$$\frac{dR_t^d}{dV_t} = \frac{1}{\beta^2}\frac{N_t\frac{dL_{it}}{dV_t}sY - sY_{tt}}{\left(sY_t - N_t L_{it} + V_t\right)^2} \qquad (2)$$

From the chain rule, we know that $\frac{dL_{it}}{dV_t} = \frac{dL_{it}}{dR_t^d}\frac{dR_t^d}{dV_t}$ which means that we can substitute (1) into (2) to get:

$$\frac{dR_t^d}{dV_t} = \frac{1}{\beta^2}\frac{N_t\left(\frac{-1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)\frac{dR_t^d}{dV_t}sY_t}{\left(sY_t - N_t L_{it} + V_t\right)^2} + \frac{1}{\beta^2}\frac{-sY_t}{\left(sY_t - N_t L_{it} + V_t\right)^2}$$

$$\Rightarrow \frac{dR_t^d}{dV_t}\left(1+\frac{1}{\beta^2}\frac{N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}{\left(sY_t-N_tL_{it}+V_t\right)^2}\right)=\frac{1}{\beta^2}\frac{-sY_t}{\left(sY_t-N_tL_{it}+V_t\right)^2}$$

which can be further simplified to the following

$$\Rightarrow \frac{dR_t^d}{dV_t}\left(\frac{\beta^2\left(sY_t-N_tL_{it}+V_t\right)^2+N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}{\left(sY_t-N_tL_{it}+V_t\right)^2}\right)=\frac{1}{\beta^2}\frac{-sY_t}{\left(sY_t-N_tL_{it}+V_t\right)^2}$$

and finally to

$$\frac{dR_t^d}{dV_t}=\frac{-sY_t}{\beta^2\left(sY_t-N_tL_{it}+V_t\right)^2+N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}<0 \qquad (3)$$

which corresponds to equation (27) in the main text.

To get the effect of a change in the future level of output from a change in bank capital (i.e. an indirect change to the level of lending) we use:

$$\frac{dY_{t+1}}{dV_t}=aAN_tL_{it}^{a-1}\frac{dL_{it}}{dV_t}$$

Or similarly, through the chain rule used above,

$$\frac{dY_{t+1}}{dV_t}=aAN_tL_{it}^{a-1}\frac{dL_{it}}{dR_t^d}\frac{dR_t^d}{dV_t} \qquad (4)$$

Plugging (1) and (3) into (4) of this supplement we get that

$$\frac{dY_{t+1}}{dV_t} = aAN_t L_{it}^{a-1}\left(\frac{-1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)\frac{-sY_t}{\beta^2\left(sY_t - N_t L_{it} + V_t\right)^2 + N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}$$

After simplifying the signs, it is evident that the first derivative is greater than zero.

$$\frac{dY_{t+1}}{dV_t} = \frac{aAN_t L_{it}^{a-1}\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}{\beta^2\left(sY_t - N_t L_{it} + V_t\right)^2 + N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t} > 0 \tag{5}$$

Here, further simplifications are appropriate. From equation (7) in the main text we have that $R_t^d = \frac{1}{\beta^2}\frac{D_t}{\Pi_t - D_t}$ or

$$R_t^d = \frac{1}{\beta^2}\frac{D_t}{\Pi_t - D_t} \Rightarrow R_t^d \beta^2\left(\Pi_t - D_t\right) = D_t \Rightarrow \beta^2\left(\Pi_t - D_t\right) = \frac{D_t}{R_t^d}$$

which after substituting in equation (18) in the main text and using the facts that $\Gamma_t = N_t L_{it}$ and t $\Pi_t = sY_t$, we get that

$$\frac{D_t}{R_t^d} = \beta^2\left(sY_t - N_t L_{it} - V_t\right) \tag{6}$$

Using (6) to simplify (5), we have that

$$\frac{dY_{t+1}}{dV_t} = \frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\frac{aAN_t L_{it}^{a-1}\left(R_t^d\right)^{\frac{-1}{1-a}-1} sY_t}{\left(D_t\right)^2\left(R_t^d\right)^{-2} + N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t} > 0$$

In addition, substituting the loan equation (22) from the main text into the above we get that,

$$\frac{dY_{t+1}}{dV_t}=\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\frac{aAN_t\left[\frac{1}{2}\frac{aA}{R_t^d}\right]^{-1}\left(R_t^d\right)^{\frac{-1}{1-a}-1}sY_t}{\left(D_t\right)^2\left(R_t^d\right)^{-2}+N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}>0$$

which simplifies to,

$$\frac{dY_{t+1}}{dV_t}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\frac{N_t\left(R_t^d\right)^{\frac{-1}{1-a}}sY_t}{\left(D_t\right)^2\left(R_t^d\right)^{-2}+N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-1}{1-a}-1}\right)sY_t}>0$$

and further to, $\frac{dY_{t+1}}{dV_t}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\frac{N_t\left(R_t^d\right)^{\frac{-1}{1-a}}sY_t}{\left(R_t^d\right)^{\frac{-1}{1-a}}\left(\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}\right)sY_t\right)}>0$

while finally we find $\frac{dY_{t+1}}{dV_t}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\frac{N_t sY_t}{\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}sY_t}>0$

which corresponds to equation (28) in the main text.

Supplement 2

Mathematical workings for the calculation of the second derivative of output with respect to a change in lending

From equation (28) in the main text, we know that

$$\frac{dY_{t+1}}{dV_t}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\frac{N_t sY_t}{\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\left(\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}\right)sY_t}>0$$

To get the second derivative, we again differentiate the above, with respect to V_t, i.e.

$$\frac{d^2Y_{t+1}}{dV_t^2}=\frac{-2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}N_t sY_t\frac{2D_t\left(R_t^d\right)^{\frac{-3-2a}{1-a}}\frac{dD_t}{dV_t}-\left[\frac{3+2a}{1-a}\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t\right]\frac{dR_t^d}{dV_t}}{\left(\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}sY_t\right)^2}$$

or, simply

$$\frac{d^2Y_{t+1}}{dV_t^2}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}N_t sY_t\frac{-2D_t\left(R_t^d\right)^{\frac{-3-2a}{1-a}}\frac{dD_t}{dV_t}+\left[\frac{3+2a}{1-a}\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t\right]\frac{dR_t^d}{dV_t}}{\left(\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}sY_t\right)^2}$$

We know from (7) in the main text that $R_t^d=\frac{1}{\beta^2}\frac{D_t}{\Pi_t-D_t}$, and thus if we differentiate it with respect to V_t we get:

$$\frac{dR_t^d}{dV_t}=\frac{1}{\beta^2}\frac{\frac{dD_t}{dV_t}\left(\Pi_t-D_t\right)-\left(-\frac{dD_t}{dV_t}\right)D_t}{\left(\Pi_t-D_t\right)^2}\Leftrightarrow\frac{dR_t^d}{dV_t}\beta^2\left(\Pi_t-D_t\right)^2=\frac{dD_t}{dV_t}\Pi_t$$

which suggests that $\frac{dD_t}{dV_t}=\frac{dR_t^d}{dV_t}\frac{\beta^2\left(\Pi_t-D_t\right)^2}{\Pi_t}$, or, after using the fact that $\Pi_t=sY_t$, it becomes

$$\frac{dD_t}{dV_t}=\frac{dR_t^d}{dV_t}\frac{\beta^2\left(sY_t-D_t\right)^2}{sY_t} \tag{1}$$

Substituting (1) into the equation for the second derivative yields that

$$\frac{d^2Y_{t+1}}{dV_t^2}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}N_t sY_t\frac{\left[-2D_t\left(R_t^d\right)^{\frac{-3-2a}{1-a}}\frac{\beta^2\left(sY_t-D_t\right)^2}{sY_t}+\frac{3+2a}{1-a}\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t\right]\frac{dR_t^d}{dV_t}}{\left(\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}sY_t\right)^2} \tag{2}$$

To check the sign of the second derivative, we need to examine the numerator of the fraction in (2), i.e.

$$-2D_t\left(R_t^d\right)^{\frac{-3-2a}{1-a}}\frac{\beta^2\left(sY_t-D_t\right)^2}{sY_t}+\frac{3+2a}{1-a}\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t$$

Or,

$$-2D_t\left(R_t^d\right)^{\frac{-3-2a}{1-a}}\frac{\beta^2\left(sY_t-D_t\right)}{sY_t}\left(sY_t-D_t\right)+\frac{3+2a}{1-a}\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t \tag{3}$$

We know again from equation (7) in the main text that $R_t^d=\frac{1}{\beta^2}\frac{D_t}{\Pi_t-D_t}=\frac{1}{\beta^2}\frac{D_t}{sY_t-D_t}$ and thus,

$$\frac{D_t}{R_t^d \beta^2} = sY_t - D_t \tag{4}$$

Substituting (4) into the numerator equation (3) yields

$$-2D_t \left(R_t^d\right)^{\frac{-3-2a}{1-a}} \frac{\beta^2 \left(sY_t - D_t\right)}{sY_t} \frac{D_t}{R_t^d \beta^2} + \frac{3+2a}{1-a} \left(D_t\right)^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} + N_t \frac{1}{1-a} \left[\frac{1}{2} aA\right]^{\frac{1}{1-a}} \left(R_t^d\right)^{-2} sY_t$$

or

$$-2D_t^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} \frac{\left(sY_t - D_t\right)}{sY_t} + \frac{3+2a}{1-a} \left(D_t\right)^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} + N_t \frac{1}{1-a} \left[\frac{1}{2} aA\right]^{\frac{1}{1-a}} \left(R_t^d\right)^{-2} sY_t \tag{5}$$

Equation (5) can be further simplified to

$$2D_t^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} \frac{D_t}{sY_t} - 2D_t^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} + \frac{3+2a}{1-a} \left(D_t\right)^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} + N_t \frac{1}{1-a} \left[\frac{1}{2} aA\right]^{\frac{1}{1-a}} \left(R_t^d\right)^{-2} sY_t$$

Using the common terms, the above becomes

$$2D_t^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} \frac{D_t}{sY_t} + \left(\frac{3+2a}{1-a} - 2\right) \left(D_t\right)^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} + N_t \frac{1}{1-a} \left[\frac{1}{2} aA\right]^{\frac{1}{1-a}} \left(R_t^d\right)^{-2} sY_t$$

while we can simplify it to

$$2D_t^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} \frac{D_t}{sY_t} + \left(\frac{3+2a-2+2a}{1-a}\right) \left(D_t\right)^2 \left(R_t^d\right)^{\frac{-4-a}{1-a}} + N_t \frac{1}{1-a} \left[\frac{1}{2} aA\right]^{\frac{1}{1-a}} \left(R_t^d\right)^{-2} sY_t$$

and further to

$$2D_t^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}\frac{D_t}{sY_t}+\left(\frac{1+4a}{1-a}\right)\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t>0 \tag{6}$$

which is positive.

Substituting (6) into the second derivative equation (2) we get that

$$\frac{d^2Y_{t+1}}{dV_t^2}=\frac{2}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}N_t sY_t\frac{\left[2D_t^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}\frac{D_t}{sY_t}+\left(\frac{1+4a}{1-a}\right)\left(D_t\right)^2\left(R_t^d\right)^{\frac{-4-a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-2}sY_t\right]\frac{dR_t^d}{dV_t}}{\left(\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+N_t\frac{1}{1-a}\left[\frac{1}{2}aA\right]^{\frac{1}{1-a}}\left(R_t^d\right)^{-1}sY_t\right)^2} \tag{7}$$

where, after taking out the common terms, we get

$$\frac{d^2Y_{t+1}}{dV_t^2}=\frac{\frac{2sY_tN_t}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{-4-a}{1-a}}\left[\frac{1+4a}{1-a}\left(D_t\right)^2+\frac{sY_tN_t}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}}\left(R_t^d\right)^{\frac{6-a}{1-a}}+2\frac{\left(D_t\right)^3}{sY_t}\right]\frac{dR_t^d}{dV_t}}{\left[\left(D_t\right)^2\left(R_t^d\right)^{\frac{-3-2a}{1-a}}+\frac{1}{1-a}\left(\frac{aA}{2}\right)^{\frac{1}{1-a}}sY_tN_t\left(R_t^d\right)^{-1}\right]^2}$$

which corresponds to equation (29) in the main text.